Noname manuscript No.
(will be inserted by the editor)

Information Design in Affiliate Marketing

Sharadhi Alape Suryanarayana · David Sarne · Sarit Kraus

Received: date / Accepted: date

Abstract The recent massive proliferation of affiliate marketing suggests a new e-commerce paradigm which involves sellers, affiliates and the platforms that connect them. In particular, the fact that prospective buyers may become acquainted with the promotion through more than one affiliate to whom they are connected calls for new mechanisms for compensating affiliates for their promotional efforts. In this paper, we study the problem of a platform that needs to decide on the commission to be awarded to affiliates for promoting a given product or service. Our equilibrium-based analysis, which applies to the case where affiliates are a priori homogeneous and self-interested, enables showing that a minor change in the way the platform discloses information to the affiliates results in a tremendous (positive) effect on the platform's expected profit. In particular, we show that with the revised mechanism the platform can overcome the multi-equilibria problem that arises in the traditional mechanism and obtain a profit which is at least as high as the maximum profit in any of the equilibria that hold in the latter.

Keywords affiliate marketing · equilibrium · dynamic pricing · mechanism design · gig economy

Sharadhi Alape Suryanarayana
Department of Computer Science
Bar-Ilan University, Israel
E-mail: sharadhi.as@gmail.com

David Sarne
Department of Computer Science
Bar-Ilan University, Israel
E-mail: david.sarne@biu.ac.il

Sarit Kraus
Department of Computer Science
Bar-Ilan University, Israel
E-mail: sarit@cs.biu.ac.il

1 Introduction

Affiliate marketing is a new e-commerce paradigm in which by promoting other people's (or companies') products or services one can earn a commission (either resulting from a click or from an actual sale) [33,42]. The idea is that content producers can monetize their network of followers by promoting products and services, hence saving manufacturers time and effort in reaching their target audience. The manufacturers in turn compensate the content producers by sharing some of the revenue and this has led to many companies nowadays offering content producers a financial incentive to promote their product through an affiliate program. The content producers are internet-based businesses that have multiple streams of income such as programmatic advertisement revenue (advertisements on websites), revenue from pre-roll advertisements (e.g., Youtube advertisements), custom content (sponsored posts, brand shoutouts etc.) and affiliate marketing [19] to monetarily support the content that, in many cases, is available at no cost to the reader. Affiliate marketing, which is an important source of revenue, has become a very large industry and a key source of online income for many thousands of professional bloggers, celebrities and social media stars, ultimately creating passive income streams.

Affiliate Marketing is a key component of an emerging trend called gig economy. Gig economy can be defined as 'people using apps (also commonly known as platforms) to sell their labour' [56] and algorithmic management is central to the operation of such online labour platforms [58]. Gig work can be local where work is transacted via platforms but delivered locally such as food delivery and couriering, or it can be remote where a wide variety of digital services such as data entry and programming are outsourced to remote workers [27].

In this sense affiliate Marketing is a source of income while working at the comfort of sitting at one's home [57] and on the other hand affiliate marketing can also be used to promote gig apps or companies that the affiliates are passionate about [14,50]. All in all, even though many consider affiliate marketing to be a non-easily comprehensible marketing strategy (compared to, for example, pay-per-click (PPC) [43] and email marketing [44]), recent projections suggest it will generate billions in revenue in the coming years.[1]

The process of affiliate marketing generally involves four parties, namely the advertiser, the platform, the affiliate and the buyer. First, there is the advertiser, who sells a product or a service online, and the affiliate who generates content through which she can promote the product. The connection between the advertiser and the affiliate is made through a platform (such as AWIN (www.awin.com), ShareASale (www.shareasale.com), Maxbounty (www.maxbounty.com), Tradedoubler (www.tradedoubler.com) and CJ Affiliate (www.cj.com)). The platform lists products and services requiring promotion and provides special links, known as affiliate links, which the affiliates use

[1] E.g., according to a Business Insider Report from 2018, affiliate marketing is projected to generate \$8.2 billion revenue in the US by 2022 [52].

for directing prospective buyers to the product web page on the advertiser's website and for identifying the affiliate (usually implemented using cookies). The platform is also the one receiving the payment from the advertiser and gets to decide how much to offer the affiliates for their service [15]. Finally, there is the buyer who is usually unaware of these dynamics that take place between the advertiser, the platform and the affiliate.

Affiliate marketing offers several benefits to the three major parties involved in the process. For the advertiser, it offers a significant decrease in advertising expenditure and is highly helpful in reaching out to new customers at a low acquisition cost [15]. Also, affiliate marketing is particularly beneficial to new websites/start-ups that cannot churn out hefty sums of money on publicising their product or service. For the affiliate, it offers an opportunity to leverage her social network for monetary gains. For the prospective buyers, affiliate marketing offers exposure to new products or services from trustworthy sources that could have gone unnoticed in the context of personalized internet advertising or any other legacy forms of advertising [41].

Most research on affiliate marketing to date has focused on the benefits and the potential of this new mechanism [11, 15], pricing strategies for the advertiser [33], methods for recommending affiliates to advertisers [38,47] and fraud detection and prevention [2, 8, 54]. None of these works has considered affiliates' strategic considerations in their decision whether or not to promote a specific product.

In this paper we provide a game-theoretic based analysis of a multi-agent system with the affiliate marketing platform and affiliates as the two distinct types of agents, when the affiliates are a priori homogeneous, i.e., they are all subject to the same posting (promoting) cost, and overlaps between their followers are drawn from a common probability distribution function. To the best of our knowledge, this is the first attempt to date to model the agents of the affiliate marketing network and to provide a formal game-theoretic based analysis of such a model. Not only did prior work not take into consideration affiliates' strategic behavior (i.e., their choice of whether or not to promote a product), it also did not take into account the network effect in the sense that affiliates may have common/overlapping followers. The analysis is initially carried out on the core mechanism which models affiliate marketing as it is done today. Our analysis reveals that in many cases there is not one, but many possible equilibria. In such cases, the core equilibrium analysis does not provide a way to determine which of the equilibria will be adopted by the agents. Hence the platform cannot optimize the mechanism, and in particular has no way to determine the optimal payment to be offered to affiliates, since the effectiveness of such payments will depend on the specific equilibrium reached by the agents. To overcome the problem, we introduce a simple information disclosure scheme called the Sequential Mechanism, enabling disclosing the number of affiliates that have already become acquainted with a given opportunity. We show that even by revealing such minimal information, the platform can enforce an equilibrium using a dynamic commission structure of the highest possible expected profit among those that hold in the legacy mechanism used in practice in most platforms nowadays.

In the following section, we review related work, primarily in the area of affiliate marketing. The model and assumptions used are formally introduced in Section 3. Comprehensive equilibrium analysis and illustrative numerical examples are provided in Section 4. Finally, we provide the discussion and conclusions in Section 5.

2 Related Work

One of the earliest reported use of affiliate marketing is CDNow's BuyWeb program launched in 1994. Being an online music retailer, CDNow offered music-oriented websites to review or list albums on their pages that their visitors might be interested in purchasing, along with a link to CDNow's page [26,50]. Since then affiliate marketing has evolved and is now a key component of internet marketing [39]. At present, Artificial Intelligence and Big Data could be used for affiliate marketing in areas such as Affiliate Recruitment, Affiliate Management, Product Data Feed Optimization, Tracking, Attribution and Forecasting [37]. Despite its high applicability [37], revenue potential [52] and presence in online markets [22], the number of studies directly touching on affiliate marketing is rather modest and it seems like research still has a long way to go in order to fully unveil the potential of this transformative mechanism. In particular there is a need for theoretical and empirical research analyzing the dynamics typically formed between all of the parties involved in affiliate marketing [16].

Prior literature has considered the conflict between the advertiser and affiliate for customer attention. For example Akçura [1] studies the incentive for a firm to join an affiliate marketing program (due to the fact that this will divert its customers to another website), pricing strategies and impact on social welfare. Bhattacharya et al [7] characterize the strategies of an advertiser and the affiliate as they compete for sponsored keywords. Libai et al [33] analyze the choice of the payment mechanisms such pay-per-click and pay-per-conversion based on the nature of relationship with the affiliate partner.

Taking the affiliate partner's perspective, Ma [36] models the decision of the partner to join an affiliate program based on parameters such as conversion rate, network traffic, opportunity costs and commission rate. However, the analysis centers around the decision of a partner to promote the product as a function of the above parameters and does not involve a strategic interaction between the different affiliate partners and the platform. On the other hand Mizuno [42] simulates affiliate and buyer agents, basing the behaviors of the agents on surveys. The simulation results attempt to suggest the right mixture of advertisement content in the affiliate's blog to increase her revenues. Benedictova and Nevosad [6], present descriptive analysis, based on surveys, from the partner's perspective and propose suggestions to the advertiser. Alas, none of the above work considers the full dynamics and inter-partner strategic considerations as in this case.

Empirical research on affiliate marketing has focused on a variety of topics such as intentions to use affiliate marketing [48], recommending affiliates to advertisers [38, 47], the effect of social media on affiliate marketing [46] and modeling the contribution of affiliate marketing to the overall revenue of a firm [4]. One interesting domain for empirical research is analyzing unethical practices by affiliates such as non-disclosure of affiliate marketing content [40] and affiliate fraud detection and prevention [2,8,54]. In order to prevent affiliate marketing abuse Edelman and Brandi analyze management structures and provide suggestions according to the scale of violation by the affiliates [17].

A strikingly similar marketing strategy and domain of research is influencer marketing where individuals/content-generating businesses that have a huge fan-following are paid for their product endorsements by the parent firm. However, the influencers are offered heterogeneous payments based on the category of followership the influencer falls into and the credibility of the endorsements from the followers' point of view while the commission offered to an affiliate follows a homogeneous structure such as pay-per-click or pay-per-conversion. Fainmesser and Galeotti [20] model the decision of an influencer with respect to allocation of sponsored and organic content based on user engagement and followership category, analyze the market implications of a change in search technology efficiency and demonstrate the effect of transparency regulation on the market for online influence in a monopoly. In an oligopolistic setting, Fainmesser and Galeotti [21] (in a companion paper) characterize the equilibrium prices for different consumer groups, characterize the inefficiencies due to consumer preferences, analyze consumer surplus and profit erosion due to competition, and also study the firms' incentive to acquire information on consumer influence. Pei and Mayzlin [49] demonstrate that the optimal level of affiliation between the firm and the influencer depends on the consumer's awareness and her prior belief on product fit. As in the case of affiliate marketing, influencer marketing has its own share of unscrupulous practices. Research in this direction has been done on identification of fake likes in Instagram [53], characterization of fake influencer accounts on Twitter [59] and helping influencers manage authenticity for themselves due to brand encroachment into their content [3].

Few somehow tangential areas of research are Cashback marketing, Referral marketing and Multi-level marketing. Cashback marketing is a form of affiliate marketing where the merchant affiliates to a site such as ebates (now named Rakuten) and customers who purchase through the affiliate site are reimbursed with a portion of transaction, called cashback. Ho et al [25] develop pricing strategies for the merchant based on consumer segments, characterize the decision of a merchant to get affiliated to a website and the impact of cashback marketing on social welfare. Zhou et al [61] determine the optimal strategies of the cashback website and the firm under different settings. However, the analysis in most of the cases is centered around the customer when determining the price as well as the cashback rate. In affiliate marketing the decision of the affiliate partner is whether or not to promote based on the social network parameters (followers and other prospective partners) and she does not determine the price that is requested from the potential buyer. Empirically, Ballestar et al [5] classify the users of a cashback website into clusters based on their commercial activity and their role within the cashback social network.

In referral programs [23, 51] whenever a consumer makes a purchase, the firm gives her a link to share with friends, and every purchase coming through that link generates a referral payment. Still, the assumptions that prior work studying referral programs models have relied on, especially those related to the nature of the promotion made, are very different than those used for affiliate marketing. For example, Lobel et al [35] assume that in the referral program, the consumer needs to directly contact each friend, i.e., incurs a cost for every referral made. Therefore, the key decision for the consumer is how many friends to contact. In affiliate marketing, the affiliate incurs a one-time publication cost for exposing all of her followers to the opportunity. Furthermore, the analysis provided by Lobel et al is based on the assumption that the population of potential consumers is represented as a rooted graph, and that only the consumer that is the root of this graph is approached by the firm. From that point on, the process follows the dynamics of a pyramid. In affiliate marketing, on the other hand, affiliates join only through accessing the platform, and there is no a priori advantage to any affiliate in the sense of getting the information before the others. Guo et al [24] investigate optimal pricing and referral reward strategies under different demand dynamics for referral marketing where an existing consumer refers the product to her friends. However, the authors do not consider the decision of the consumer on whether or not to promote the product. They instead model the probabilities of buying and referring as an effect of the price and referral reward.

With respect to multi-agent systems, research on information propagation through social networks where each node is an agent that holds some influence on the mechanism has been carried out in the form of designing mechanisms such that every agent that is buying the product is incentivized to share the information to the neighbours in her social network [32, 60]. These were used primarily for studying formal mechanisms and properties of multi-level marketing (MLM) through social influence networks [18] and the management of early adopters, taking into account their social influence, when launching a new product [12, 30]. However, none of these take into consideration the fact that the targeted buyers, followers in the case of our problem, can be connected to more than one agent that is propagating the information for a monetary gain.

Finally, with the hype of gig jobs, much general work can be found on that topic. In particular, research on manual gig work has been conducted on topics such as usage of urban taxis for food delivery [34], crowdsourcing last mile delivery by exploiting a social network of the customers [13], providing the optimal rider schedules for vehicles to maximize the overall utility in the case of ridesharing [10] and allocating spatial tasks to crowdsourced workers such that the overall cooperation is maximized [9]. In the case of gig work with digital labor, research has been conducted on verifying the quality of workers on platforms such as Amazon Mechanical Turk [28], grouping workers into tasks such that the total profit of the tasks is maximized [55] and identifying and classifying malicious tasks (also called crowdturfing tasks) on platforms such as Fiverr [31]. Unfortunately none of the above is transferable to the study of the questions that are the locus of the current paper.

3 Model

Our model considers an affiliate marketing platform and N prospective affiliate partners (denoted "partners" henceforth). The platform offers a product or a service requiring promotion and provides its affiliate link, which the partners can use for directing their followers (connections/friends who are the potential buyers) from their social network to the product's web page on the advertiser's website. The model assumes that the partners are a priori homogeneous, in the sense that each of them is connected to k followers[2] and each of the latter is potentially connected to $1 \leq w \leq N$ partners overall, where w is a priori unknown and characterized by a probability function $p_W(w)$ such that $\sum_{w=1}^{N} p_W(w) = 1$ (see Figure 1).

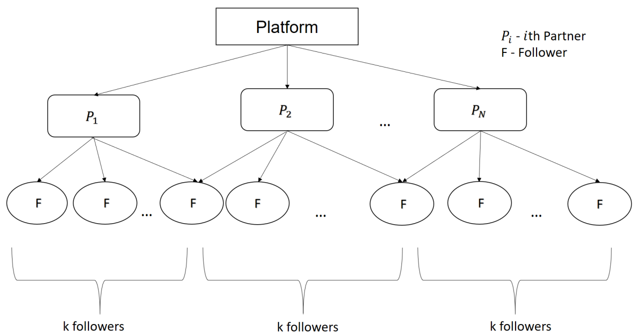


Fig. 1: Network Structure - each partner has k followers with some overlap.

A partner can either promote the product or opt not to promote it (see Figure 2 for an illustrative summary of the process). Promoting incurs a cost c, whereas not promoting does not incur any cost.[3] The model assumes that upon

[2] Alternatively, we can assume that the number of potential followers of each partner is a priori probabilistic. This would require some technical changes in the analysis, primarily adding expectation calculations in the equations, however will not change the claims and proofs.

[3] All partners are characterized with the same promotion cost as this is usually the (quite standard) cost of time it takes for uploading a post or the reputation loss associated with promoting the product in the generated content.

promoting, the content will reach all of the partner's followers, and even if a follower does not read it right away she will review the promotions in the order received (e.g., in case she is connected to several partners that promote the product). In order to encourage partners to promote the product, the platform offers a commission (in the form of a fixed payment) M for each lead that will convert to a purchase. Since followers are human, their purchase decisions are modeled as probabilistic and do not reflect any game-theoretic considerations. Therefore, we assume that a random follower will be interested in the promoted product (to the level of purchasing it) with probability p_B.[4] A follower will not use the affiliate link in case another link to the same product has already been received from one of the other partners to whom she is connected.[5] Finally, the platform's gain from each successful sale of the product is denoted G.

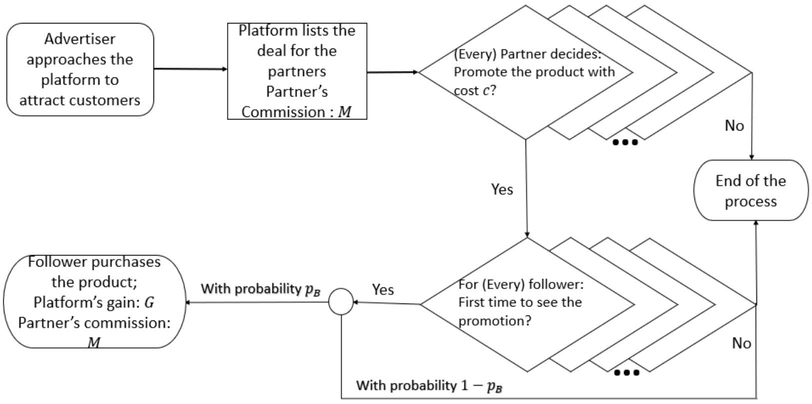


Fig. 2: The choices of partners and followers in the model, and the resulting payments and costs incurred.

All of the agents (platform and partners) are assumed to be fully rational and self-interested in the sense that their goal is to maximize their individual expected profit. This is primarily because these are internet-based businesses and their decision-making process is of a repeated nature.[6] The expected profit of the platform is its gain from successful sales minus the commissions paid to partners. The expected profit of a partner is zero if not promoting the product

[4] Notice that by setting $p_B = 1$ the model changes into (and the analysis and all proofs become applicable to) a pay-per-lead scheme.

[5] While we rely on the assumption of homogeneous partners where a follower is indifferent between the partner whose affiliate link she clicks, an alternate scenario is however possible where the follower chooses from the partner that is the most reputed/reliable according to the follower. In such cases the equilibrium computation becomes combinatorial due to the assignment of different probability of purchase for each follower and does not add much in terms of insights.

[6] Even the partners need to make promotion decisions on a daily basis driven by the above mentioned considerations of time spent, reputation loss and commissions to be earned.

and otherwise it is the expected total commissions received minus the cost of promotion. Finally, the model assumes that all of the agents are familiar with the parameters M, p_B, N, k, c, G and the function $p_W(w)$. This usually holds in real-life settings, as most of this information is either publicly available or can be found with minimal effort.[7] For exposition purposes we will refer to specific settings onward using the tuple $(N, k, M, c, G, p_B, p_W(w))$

4 Analysis

We begin with analyzing the affiliate marketing mechanism in its core form, i.e., the platform advertises the listing with the pre-set commission, M (see in Figure 2) and does not disclose any other information. We refer to this mechanism, as applied by the platforms nowadays, the Core Mechanism. We then propose and analyze a slightly modified mechanism in which the number of partners that have already viewed the listing is disclosed to the partner that views the listing while the commission offered is the same for every partner. This is referred as the Sequential Mechanism. Finally, we extend and improve the Sequential Mechanism with a Dynamic Commission Structure. Here, in addition to revealing the number of partners that have viewed the listing, we also have a commission structure that is correlated to the position of the partner in the sequence. Details of the three mechanisms are explained in the sections that follow. The profit of the platform depends on the equilibrium adopted by the partners. Hence, the analysis involves the characterization of the equilibrium of the partners and the calculation of the profit of the platform based on the equilibrium adopted by the partners. Synthetic numerical examples are used, whenever applicable, for illustration purposes.

4.1 Core Mechanism

When the platform simply lists the details of the product and generates affiliate links to partners, the setting can be considered a simultaneous game. Meaning that even though the partners do not necessarily all access the listing at the same time, they are unaware of how many of the other partners will decide to promote it.

A partner's strategy can thus be captured by the probability $0 \leq p \leq 1$ she will choose to promote the product. A pure-strategy Nash equilibrium is thus one where each partner P_i is using $p_i \in \{0, 1\}$. A mixed-strategy Nash equilibrium is the one where at least one partner P_i is using $0 < p_i < 1$. One natural and highly intuitive equilibrium (since all of the partners operate in similar conditions and are familiar with the parameters) that always holds is the symmetric equilibrium when all partners are using the same strategy p [45].[8] For

[7] For example, it is easy to know how many readers a blog post has reached [42] or to predict exposure of future posts.

[8] For example, Lobel et al use this kind of equilibrium in their referral-programs based model [35].

exposition purposes, we use this equilibrium whenever considering a mixed-strategy equilibrium, alas the proofs we provide regarding the dominance of the equilibrium of the proposed modified mechanism over mixed-strategy equilibria of the core mechanism hold also for all types of mixed-strategy equilibria.

Let us consider the partners P_1 and P_2 from Figure 1 promoting the product. While counting the number of unique followers exposed to the promotion, we need to account for the fact that P_1 and P_2 have one follower in common and each of them has a probability of 1/2 to be the one benefiting from the promotion to that follower. The generalization of this scenario to the case where the number of followers exposed to the promotion when i out of N partners promote the product, denoted $Expose(i)$, is given by:

$$Expose(i) = ik \sum_{w=1}^{N} p_W(w) \cdot \sum_{z=max(0,w-N+i-1)}^{min(w-1,i-1)} \frac{\binom{i-1}{z}\binom{N-i}{w-1-z}}{\binom{N-1}{w-1}} \frac{1}{z+1} \quad (1)$$

Here, for all of the i promoting partners, we iterate over each of the k followers in order to account for the overlap of a given follower with other promoting partners. Every follower is connected to at least one and at most N promoting partners. Hence the iteration is given by the index w which ranges from 1 to N. Since the partner under consideration is herself promoting the product we need to account for every subset of the other $w-1$ partners, promoting the product. For every subset of size z of $w-1$ partners connected to a given follower, we calculate the probability that the z partners are a part of the $i-1$ promoting partners and $w-1-z$ of them are a part of the $N-i$ partners that do not promote. Hence the probability of having z of the other $i-1$ promoting partners be part of the set of $w-1$ other partners to whom the current follower is connected is given by $\frac{\binom{i-1}{z}\binom{N-i}{w-1-z}}{\binom{N-1}{w-1}}$. Given a fixed value of w the minimum number of promoting partners is 0 and the maximum number of promoting partners is $w-1$ and the limits for z are initially set from 0 to $w-1$. However, considering the numerator of the probability, we have $z < i-1$ and $w-1-z < N-i$. Hence the range of z is $max(0, w-N+i-1)$ to $min(w-1, i-1)$. Finally, since we repeat the calculation for each of the $i-1$ partners, we eliminate the redundancy in the number of followers by dividing by $z+1$. This can be understood as each partner having a chance of $1/(z+1)$ to be the one benefiting from the promotion to that follower.

Figure 3 depicts the value of $Expose(i)$ as a function of the number of promoting partners, i for the setting $(N = 25, k = 50, p_W(w) = 1/N \ \forall w \in 1...N)$). Here, we can observe that $Expose(i)$ increases at a decreasing rate in i, which is explained by the fact that we are dealing with a fixed population of followers, with many of them being connected to more than one partner.The upper bound for the number of followers i.e the total population size, S, which is given by Equation 2 can be obtained by substituting $i = N$ (since the total number of partners cannot exceed N) in Equation 1 and simplifying further.

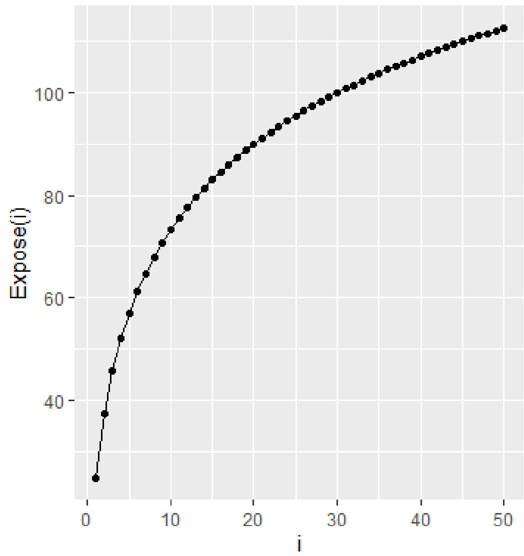


Fig. 3: $Expose(i)$ as a function of the number of promoting partners, i.

$$S = N \cdot k \cdot \sum_{w=1}^{N} \frac{p_W(w)}{w} \tag{2}$$

Hence as the number of partners increases the overlap between the followers of a newly added partner and followers of all of the other partners promoting prior to the newly added partner increases. Consequently, the number of unique followers added by the newly promoting partner decreases with every increase in the number of promoting partners.

The expected profit of a promoting partner if the total number of promoting partners (including herself) is i, denoted $B^P_{core}(i)$, is thus:

$$B^P_{core}(i) = -c + \frac{Expose(i) M p_B}{i} \tag{3}$$

Similarly, the expected profit of the platform when the total number of promoting partners is i, denoted $B^{platform}_{core}(i)$, is:

$$B^{platform}_{core}(i) = Expose(i)(G - M)p_B \tag{4}$$

The analysis of the symmetric mixed strategies is quite similar. The expected profit of the platform when the partners use a mixed strategy p, denoted $B^{platform}_{core}(p)$, is:

$$B^{platform}_{core}(p) = \sum_{i=0}^{N} \binom{N}{i} p^i (1-p)^{N-i} Expose(i) \cdot (G - M)p_B \tag{5}$$

Similarly, the expected profit of a promoting partner, if all other partners are promoting with probability p, denoted $B^P_{core}(p)$, is:

$$B^P_{core}(p) = -c + \sum_{j=0}^{N-1} \binom{N-1}{j} p^j (1-p)^{N-j-1} \frac{Expose(j+1)Mp_B}{j+1} \quad (6)$$

An alternative calculation of $B^P_{core}(p)$ is the one which considers the number of other partners to whom each of her followers is connected:

$$\begin{aligned} B^P_{core}(p) &= -c + k\sum_{w=1}^{N} p_W(w) \cdot \sum_{z=0}^{w-1} \binom{w-1}{z} \frac{p^z(1-p)^{w-z-1}}{z+1} Mp_B \\ &= -c + k\sum_{w=1}^{N} p_W(w)\frac{1}{w} \cdot \sum_{z=0}^{w-1} \binom{w}{z+1} p^z(1-p)^{w-z-1} Mp_B \\ &= -c + k\sum_{w=1}^{N} p_W(w)\frac{1}{w} \cdot \sum_{z=1}^{w} \binom{w}{z} p^{z-1}(1-p)^{w-z} Mp_B \\ &= -c + k\sum_{w=1}^{N} p_W(w)\frac{1}{wp} \cdot \sum_{z=1}^{w} \binom{w}{z} p^{z}(1-p)^{w-z} Mp_B \\ &= -c + k\sum_{w=1}^{N} p_W(w)\frac{(1-(1-p)^w)}{wp} Mp_B \end{aligned} \quad (7)$$

Here we iterate over the k followers of the partner, considering for each of them the number of partners w to whom she is connected. For each w we calculate the probability that z of the $w-1$ other partners are promoting (given that each one promotes with probability p), in which case the partner has a chance of $1/(z+1)$ to be the one benefiting from the promotion to that follower.

Figure 4 depicts the value of $B^P_{core}(p)$ as a function of the probability of promotion, p for the setting $(N = 25, k = 50, M = 9, c = 4, p_B = 0.1, p_W(w) = 1/N \ \forall w \in 1...N))$. Here, we can observe that $B^P_{core}(p)$ decreases with an increase in p.

Characterization of the equilibrium. The best response strategy of every partner is: (a) to promote if $B^P_{core}(i+1) \geq 0$ whenever the other partners are using a pure strategy according to which i of them promote; and (b) to promote if $B^P_{core}(p) \geq 0$ whenever the other partners are promoting with probability $0 < p < 1$. A pure-strategy Bayesian Nash Equilibrium (BNE) solution where $i \leq N$ partners promote is thus one where $B^P_{core}(i) \geq 0 \geq B^P_{core}(i+1)$, i.e., neither promoting nor non-promoting partners have an incentive to deviate. A symmetric BNE solution p is one where $B^P_{core}(p) = 0$. The above analysis can be augmented to accommodate solutions where some of the partners use pure strategies and some use mixed ones. The extension is quite mathematically technical and does not contribute much in terms of results, therefore it is omitted.

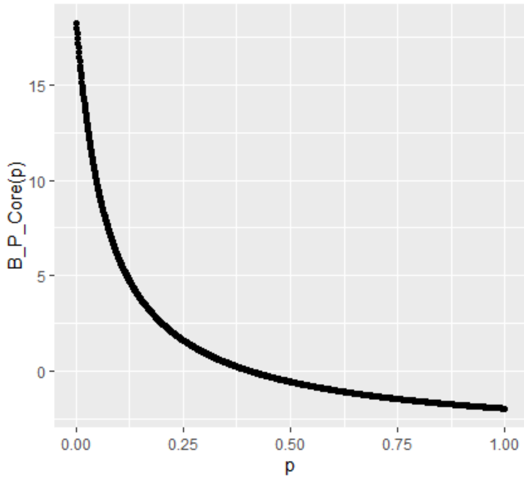


Fig. 4: $B^P_{core}(p)$ as a function of the probability of promotion, p.

Proposition 1 *Any i-based promoting partners pure-strategy equilibrium of the core mechanism in which the partners are making zero profit (i.e., $B^P_{core}(i) = 0$ according to (3)) results in at least as high expected profit (to the platform) as any other i-based pure-strategy equilibrium in which the partners' expected profit is positive.*[9]

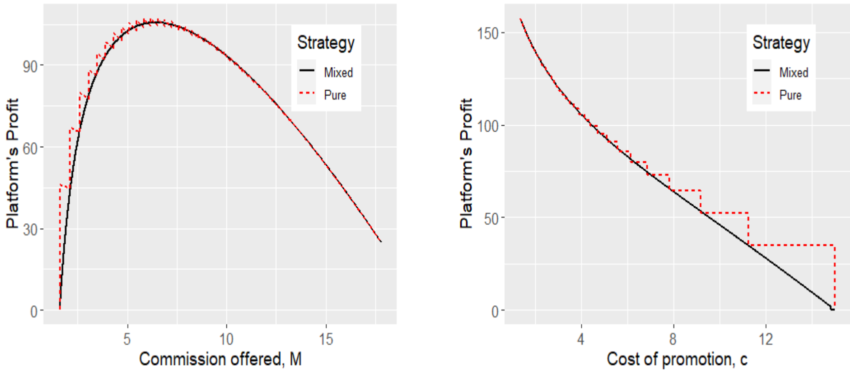
Proof. This derives from the fact that the platform's gains from the partners' promotions are the same (and equal $Expose(i) \cdot p_B \cdot G$), whereas in the case where partners' profit is zero, the expected sum of commissions paid is necessarily lower than when they make a profit (as the partners' gain derives solely from the platform's commission). $\square$

Based on Proposition 1, we can calculate the expected-profit-maximizing M value by setting $B^P_{core}(i) = 0$ and solving (3) for any i (and calculating the corresponding expected profit $B^{platform}_{core}(i)$ according to (4)). Now, all we need to do is go over the N latter values ($B^{platform}_{core}(i)$, $\forall i$) and pick the M value associated with the maximum among them.[10]

Figure 5 depicts the platform's profit with the mixed and pure equilibria as a function of the commission used, M, and of the cost of promotion, c, for a setting ($N = 50, k = 25, M = *, c = *, G = 20, p_B = 0.1, p_W(w) = 1/N \ \ \forall w \in 1...N$). For the graph that uses the commission M as the independent variable (left), the cost of promotion is set as $c = 4$ and for the graph that uses the

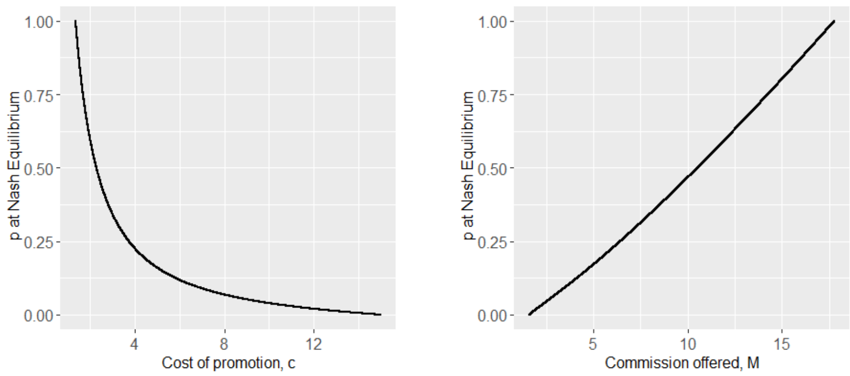
[9] An i-based promoting partners pure equilibrium necessarily exists for any $i > 0$, as the increase in $Expose(i)$ due to an increase in i is a decreasing function. Figure 3 which depicts $Expose(i)$ as a function of the number of promoting partners, i, visualizes this assertion.

[10] The calculation for the M value that maximizes the platform's expected profit with mixed strategies is more complex, yet as we prove later on it is unnecessary as it is dominated by a pure-strategy equilibrium.



(a) (b)

Fig. 5: Influence of commission M and cost of promotion c on profit.

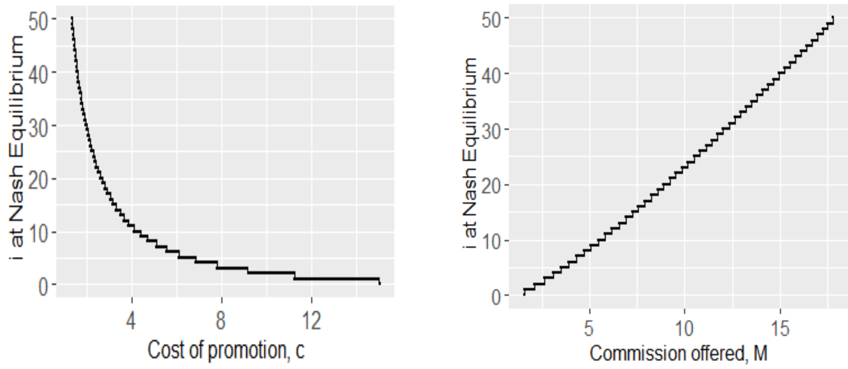


(a) (b)

Fig. 6: Influence of cost of promotion c and commission M on the mixed-strategy equilibrium.

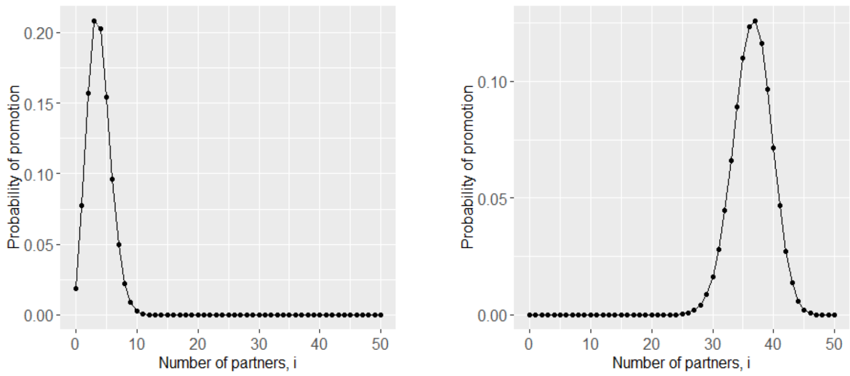
promotion cost as the independent variable (right) the commission is set as $M = 6$. In the left graph we observe that while the mixed-strategy equilibrium is continuous in M (as the probabilities used by the partners are continuous), the pure-strategy equilibrium exhibits a recurring pattern of a sharp increase (a step-function) followed by a continuous decrease. The increase is associated with a transition from an equilibrium based on i promoting partners to one with $i + 1$. The decrease is when the number of promoting partners remains the same, yet the increase in M reduces the profit gained from any purchasing follower. This provides the intuition for the proof of Proposition 1.

Further, we observe from Figure 5 that with small M values, the pure-strategy equilibria generally yield a greater expected profit compared to the mixed ones (for the same M value) and very similar profit when M is relatively



(a) (b)

Fig. 7: Influence of cost of promotion c and commission M on pure-strategy equilibrium.



(a) (b)

Fig. 8: Influence of mixed strategy equilibrium on number of promoting partners: (a) Probability distribution of the number of promoting partners when $N = 50$, $p = 0.076$; and (b) Probability distribution of the number of promoting partners when $N = 50$, $p = 0.73$.

high. This is explained by the nature of the induced number of promoting partners probability function. When M is small, the promotion probability is relatively small, and since $Expose(i)$ increases at a decreasing rate the effect of those cases where the number of promoting partners with mixed strategies is smaller than with pure strategies is significantly greater than with the opposite cases. When M is high, the promotion probability is high hence the differences between the above cases are very small.

Figures 6 and 7 show the influence of the cost of promotion, c and commission, M, on the values of the probability of promotion in equilibrium when the partners are using mixed strategies and the number of promoting partners in equilibrium when the partners are using pure strategies respectively. For a given value of p, $B^P_{core}(p)$ is directly proportional to the value of M. An increase in M, therefore, increases the value of p at which $B^P_{core}(p) = 0$. The smaller the value of p, the more skewed the distribution (of the number of promoting partners) is to the right, and the higher the value of p, the more skewed the distribution is to the left. This is due to the fact that the distribution of the number of promoting partners follows a binomial distribution and the pattern of skewness is in accordance with the binomial distribution. Figure 8 illustrates the influence of the mixed-strategy equilibrium on probability (of the binomial distribution) of the number of promoting partners. The values of p considered are the mixed strategy equilibrium at $M = 3.167$ and $M = 13.948$. The same can be said about the influence of M on $B^P_{core}(i)$ and the value of i at equilibrium as M increases. However, instead of a continuous increase as in the case of the partners using mixed strategies, the value of i at equilibrium exhibits an increasing step function with the increase in M.

Hence, when M is relatively small, p is relatively small and the distribution of the number of promoting partners with the mixed equilibrium is right-skewed (Figure 8a). Therefore, the loss due to cases where a relatively small number of partners promote is substantial. When M is relatively high, the distribution of promoting partners is left-skewed (Figure 8b), therefore the loss due to cases of small number of promoters diminishes while the gain from better using M to induce higher promotion becomes more apparent. As expected, overall, the expected profit curve generally increases and, upon reaching a maximum level, decreases. This is because with small commissions, the number of promoting partners is small (with pure-strategy equilibrium) and their distribution is skewed to the right (with mixed equilibrium) hence any increase in M will strongly affect the number of unique followers reached. The decrease with small M values is explained by the reversed effect.

The cost of promotion, c, is negatively proportional to the expected profit i.e. an increase in the cost of promotion leads to a decrease in the expected profit (Figure 5b). However, the comparison of the pure strategy equilibrium with the mixed strategy equilibrium follow similar dynamics as in the case of the influence of the commission offered, M. For a given value of p or i, an increase in the value of c results in the decrease of $B^P_{core}(p)$ or $B^P_{core}(i)$ respectively. Hence, an increase in c, decreases the value of p or i at which $B^P_{core}(p) = 0$ and $B^P_{core}(i) = 0$. The only difference between the effect of M and c is in the general trend—unlike with the increase in the commission used, the increase in the cost of promoting always results in a general decrease in the number of promoting partners, or to a more skewed distribution of promoting partners with mixed equilibrium, hence the expected profit generally decreases.

Next we show the effect of the increase in the number of followers (per partner, i.e., the followers' population is not pre-set and its size changes (increases) as the number of followers each partner has increases) on the resulting equilibria. Taking all other model parameters to be fixed, an increase in the number of followers will necessarily result in an increase in the platform's expected profit. Figure 9 illustrates the phenomenon. The setting used is ($N = 50, k = \{5, ..., 112\}, M \in \{4, 7\}, c = 4, G = 20, p_B = 0.1, p_W(w) = 1/N \;\; \forall w \in 1...N$). Graphs (a) and (b) of the figure depict the platform's expected profit, for $M = 4$ and $M = 7$ (where the first corresponds to an interval where the increase in M results in an increase in expected profit, and the second to an interval associated with a decrease - see Figure 5a which illustrates the influence of the commission on the platform's profit). As expected, the resulting increase in the size of the population yields an increase in the platform's expected profit. While the difference between the platform's profit while using pure and mixed strategies is not easily distinguishable, the more interesting influence is that of the influence of the commission, M on the probability of promotion in equilibrium. Graph (c) in the figure depicts the promotion probability of the partners in equilibrium. Here, the increase in M results in a greater promotion probability, for every number of followers. Consequently, mixed-strategy equilibria cease to hold with a smaller number of followers. Figure 10 provides a similar perspective, one that varies the value of the promotion cost c. It uses the exact same setting, this time however with $k = \{9, ..., 124\}$, $M = 9$ and $c \in \{9, 10\}$. While the general pattern is similar to the one exhibited in Figure 9, the effect of the increase in c is reversed - an increase in c results in a decreased promotion equilibrium probability.

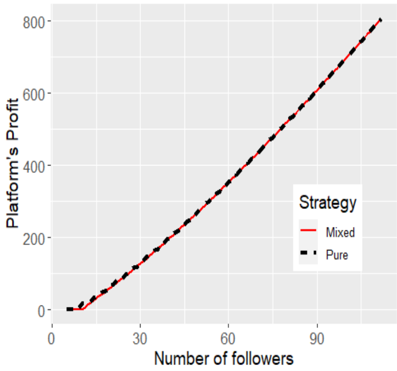
All in all, we observe that neither of the equilibria types (pure and mixed) generally dominate the other in terms of the platform's expected profit for any given M. Later on, we prove that if M is within the control of the platform, as is often the case in most real-world settings, the pure-strategy equilibrium yields the maximum expected profit for the platform.[11]

4.2 Proposed Modified Mechanism

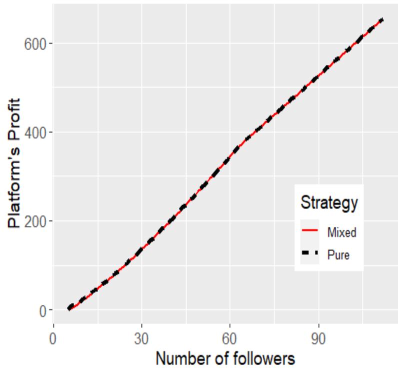
While with the core mechanism the platform cannot influence the equilibrium that will hold in a multi-equilibria scenario, we propose a simple modification to the mechanism. With the modified mechanism only one equilibrium holds. The idea is that the platform will provide every partner that accesses the listing with the number of other partners that have already accessed it (see Figure 11).[12] This small change turns the game into a sequential one, as it enables each partner some additional information. Therefore, while in the core mechanism a pure strategy of a partner is simply a binary decision - whether or not to promote, here the strategy of partner P_i is a *function*, determining whether or not to promote given the information about the number of other partners that have accessed the listing so far.

[11] An example where M is not fully within the control of the platform is when the platform offers a fixed M for all products or services listed on its website.

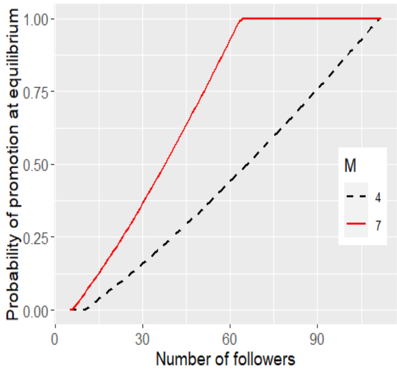
[12] Meaning that we do not even need to provide information about how many others have received an affiliate link. Instead we only provide information about how many times the listing was uniquely viewed.



(a)



(b)



(c)

Fig. 9: The platform's expected profit for pure and mixed strategies equlibria as a function of the number of followers for $M = 4$ (a), $M = 7$ (b), and the promotion probability used (for mixed equilibria) as a function of the number of followers (c).

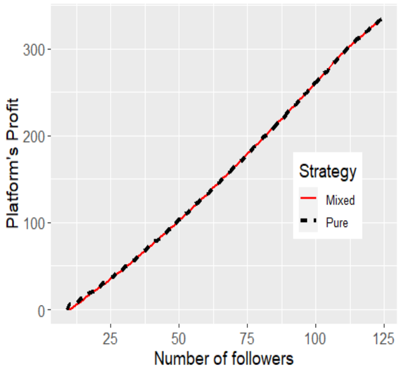
The modified mechanism (denoted "Sequential" henceforth) is a dynamic game. As such, the proper, or natural solution concept for this model is the sub-game perfect Nash equilibrium (SPNE). We note that since the sequential game is a game of complete information, the SPNE can be easily computed using backward induction. Note that except for the relatively rare case of ties, the sub-game perfect Nash equilibrium is unique and uses pure strategies.

We use $Expose^{marginal}(i)$ to denote the expected number of new followers becoming exposed to the product as a result of the ith promotion. Formally,

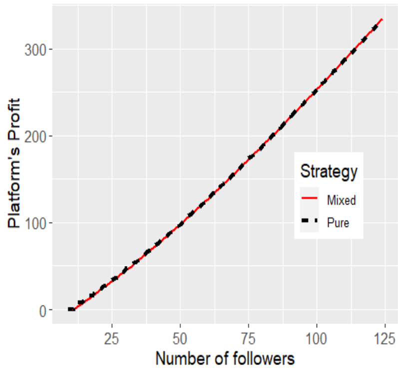
$$Expose^{marginal}(i) = Expose(i) - Expose(i-1) \tag{8}$$

The expected profit of a partner who is the ith promoting partner is therefore:

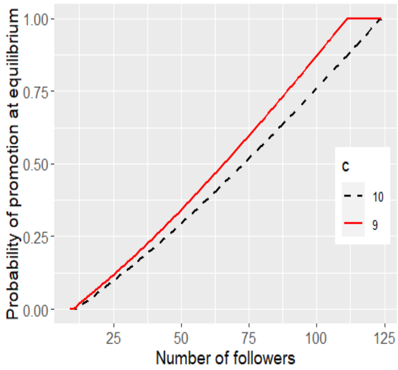
$$B^P_{seq}(i) = -c + Expose^{marginal}(i) M p_B \tag{9}$$



(a)



(b)



(c)

Fig. 10: The platform's expected profit for pure and mixed strategies equlibria as a function of the number of followers for $c = 9$ (a), $c = 10$ (b), and the promotion probability used (for mixed equilibria) as a function of the number of followers (c).

Proposition 2 *The SPNE for the sequential mechanism is to have each partner participate if the number of other partners who accessed the listing is* $n' < n^*$*, where* $n^* = \lfloor n \rfloor$ *such that* n *is the solution to* $B^P_{seq}(n) = 0$ *(according to* (9)*), and avoid participation otherwise.*

Proof. Consider any partner who receives information $n' < n^*$. In this case, the expected profit if promoting is at least $B^P_{seq}(n'+1)$, as $B^P_{seq}(i)$ decreases in i[13] and the number of promoting partners so far is at most n'. Since $n'+1 \leq n^*$ and $B^P_{seq}(i)$ decreases in i, $B^P_{seq}(n'+1) \geq B^P_{seq}(n^*) \geq 0$. Therefore promoting is the dominating strategy and as such all partners receiving information $n' < n^*$ will promote. Now consider a partner receiving information $n' \geq n^*$. Knowing that the first n^* partners who viewed the listing necessarily promoted it, the

[13] Since $Expose(i)$ increases at a decreasing rate in i, $Expose^{marginal}(i)$ decreases in i.

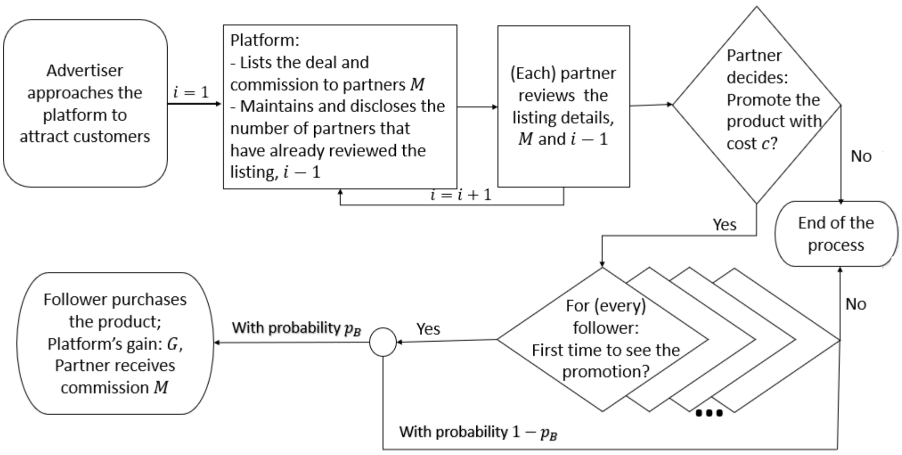


Fig. 11: Pictorial representation of the Sequential Mechanism.

partner will find not promoting to be the dominating strategy, as $B^P_{seq}(n') \leq B^P_{seq}(n^*+1) < 0$. □

The expected profit of the platform can be calculated using 4, substituting $i = n^*$.

Unfortunately, the sequential mechanism as presented above falls behind pure-strategy Nash equilibria of the core mechanism, as stated in Proposition 3.

Proposition 3 *For any M, the SPNE of the sequential mechanism is weakly dominated by at least one pure Nash equilibrium of the core mechanism that uses the same M.*

Proof. In the sequential case in equilibrium there are exactly n^* partners choosing to promote. We show that any pure equilibrium that holds with the core mechanism is based on at least n^* promoting partners.

Consider the n^*th promoting partner in the sequential mechanism. The expected profit of this partner $B^P_{seq}(n^*)$ is non-negative, based on Proposition 2. Furthermore, $B^P_{seq}(n^*) = -c + Expose^{marginal}(n^*)Mp_B = -c + (Expose(n^*) - Expose(n^*-1))Mp_B < -c + Expose(n^*)Mp_B/n^* = B^P_{core}(n^*)$.

The interpretation of this inequality is as follows. $Expose^{marginal}(n^*)$ denotes the number of unique followers exposed to the product as a result of the n^*th promotion. $\frac{Expose(n^*)}{n^*}$ equally splits all of the followers that are exposed to the promotion among the n^* partners that are promoting the product. Since $Expose(i)$ increases at a decreasing rate (and not at a uniform rate) in i, the number of unique followers exposed to the product as a result of the n^*th promotion is less than the share of followers if we were to split $Expose(n^*)$

equally among all of the n^* partners. Therefore $B^P_{core}(n^*) > 0$, meaning that there are at least n^* partners promoting in any pure-strategy equilibrium in the core model. Therefore, since the platform's expected profit increases in the number of followers being exposed to the product (as both in the core and sequential mechanisms the same M is used), the pure-strategy equilibrium with the core mechanism offers at least the same expected profit as the SPNE of the sequential mechanism. □

4.3 Using Dynamic Commission

Proposition 3 suggests that when the commission offered is fixed, other than solving the multi-equilibria problem the sequential mechanism offers no advantage, e.g., in terms of the platform's expected profit. Fortunately, the sequential mechanism can be further revised in a way that its SPNE will yield the same expected profit as the equilibrium associated with the maximum expected profit in the core mechanism. This is achieved by replacing the fixed commission M with a changing commission ("dynamic commission"), such that the ith approaching partner will receive a commission M_i $\forall i \leq N$ (see Figure 12). By properly setting the dynamic commission, the platform can take over the entire partners' surplus.

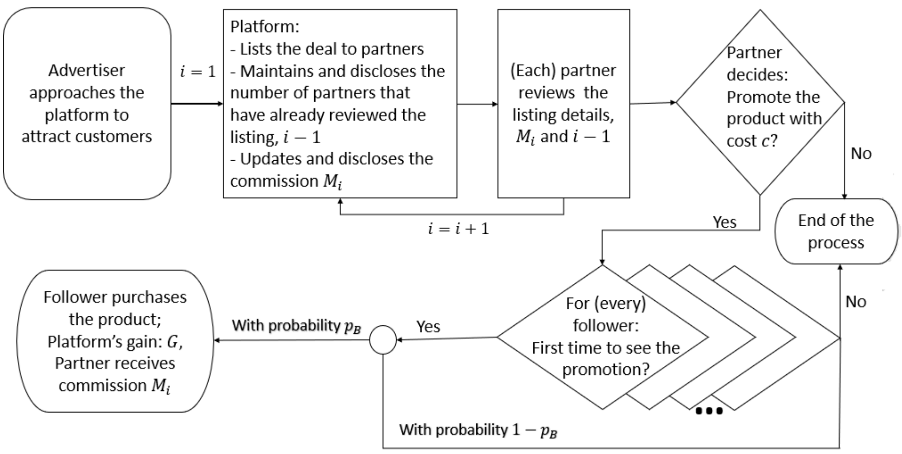


Fig. 12: Pictorial representation of the Sequential Mechanism with a Dynamic Commission Structure.

Theorem 1 *The sequential mechanism with dynamic commission will result in an SPNE with the maximum expected profit to the platform when setting the commission for the ith querying partner to* $M_i = min(\frac{c}{Expose^{marginal}(i)p_B}, G)$.[14]

Furthermore, the platform's expected profit with this SPNE will be at least as high as with any pure-strategy Nash equilibrium in the core mechanism. In particular, when the platform has full control over M in the core mechanism, the expected profit with the SPNE of the sequential dynamic mechanism will be equal to the expected profit obtained with the pure-strategy Nash equilibrium yielding the maximum expected profit in the core mechanism.

Proof. First we prove that offering a commission $min(\frac{c}{Expose^{marginal}(i)p_B}, G)$ to the ith querying partner is optimal in the sequential mechanism. Assume otherwise, i.e., the platform uses a different commission structure. Obviously, any commission structure offering partner i a commission $M_i > G$ is not optimal. This is because by using $M_i = G$ and restructuring the remaining $N-i$ commissions the platform gains twice: First, it gains from the reduction in M_i—the only possible change in partner P_i's strategy is deciding not to promote in case she was promoting with the former scheme (as the commissions offered decreases), hence the platform will avoid the loss $M_i - G$. As for the remaining $N-i$ partners, the strategic situation for the platform has improved - having P_i opt not to promote increases the expected profit of each of the other partners if choosing to promote, though the platform can extract at least the same expected profit from these partners by optimizing their commissions.

Therefore, we only consider cases where a commission other than $M_i = \frac{c}{Expose^{marginal}(i)p_B} < G$ is used. Consider the first partner P_i for which a commission $M_i' < M_i = \frac{c}{Expose^{marginal}(i)p_B}$ (and yet $M_i' < G$) is used. Since every partner $P_{j<i}$ encountered earlier in the sequence has been offered a commission M_j, her best-response strategy is to promote (since the gain from the promotion is equal to the cost of promotion (see Equation 9)). Hence all prior $i-1$ partners have promoted. Therefore P_i's best response is not to promote. Now replace the commission offered to any partner $P_{j>i-1,j\neq N}$ by the commission offered in the original scheme to partner P_{j+1}. The strategic situation for partners $P_i, ..., P_{N-1}$ is the same as the strategic situation for partners $P_{i+1}, ..., P_N$ with the original commission scheme (given that P_i opts not to promote with the original scheme). Hence the platform's expected profit remains the same. At this point the platform can potentially further improve its expected profit, by optimizing the commission to P_N. Therefore the proposed revised scheme weakly dominates the original one.

Now consider the first partner P_i for which a commission $M_i' > M_i = \frac{c}{Expose^{marginal}(i)p_B}$ (and yet $M_i' < G$) is used. Here, reducing the commission

[14] To be completely accurate, any other commission function according to which a subset of j arriving partners, where j is the integer part of the solution j' to $\frac{c}{Expose^{marginal}(j')p_B} = G$, are being offered $\frac{c}{Expose^{marginal}(l)p_B}$ (where l is their order of arrival within the sequence) and the remaining partners being offered zero will result in the same expected profit in its SPNE.

to M_i will not change the decision of P_i, as her gain remains non-negative. Consequently, none of the remaining partners will change their promotion decision and the platform's gain will increase. Therefore, a commission scheme different than the one given in the theorem is necessarily not optimal.

Next, we show that for any pure-strategy equilibrium with i promoting partners (that results in zero profit, as only equilibria of these types need to be considered according to Proposition 1) in the core mechanism, the platform can achieve the exact same expected profit in the sequential mechanism by using commissions $\frac{c}{Expose^{marginal}(i)p_B}$ for the first i participating partners and G for the remaining ones. The expected profit from those receiving a commission $\frac{c}{Expose^{marginal}(i)p_B}$ is the same as with the core mechanism as in both cases we obtain i promoting partners and the commission they receive equals their costs. The expected profit from those offered a commission G is zero. The expected profit with the sequential mechanism can be further improved by switching to the commission structure dictated by Theorem 1, as we are either giving up on partners yielding a negative expected marginal profit (when $\frac{c}{Expose^{marginal}(i)p_B} > G$) or adding partners yielding a positive profit (when $\frac{c}{Expose^{marginal}(i)p_B} < G$). This, together with Proposition 1, suggests that the sequential mechanism will result with at least the same expected profit as the core mechanism.

Finally, we show that when the expected-profit-maximizing M value is used in the core mechanism, the resulting pure-strategy equilibrium yields the same expected profit as the profit obtained with the sequential model. This derives from the fact that in the core mechanism for any $i \leq N$ there is a pure-strategy equilibrium in which there are i partners promoting (and the others do not) where the expected profit of all partners is zero. Therefore, if picking the M value that results in i-promoting-partners equilibrium such that i is the highest number of partners for which $\frac{c}{Expose^{marginal}(i)p_B} < G$ in the sequential mechanism, we get the same number of promoting partners with both mechanisms and the expected sum of commissions paid in both equals $c \cdot i$. Therefore the platform's expected profit in both is equal. $\square$

The expected profit of the platform is therefore (setting the commission using Theorem 1):

$$B_{seq}^{platform} = \sum_{i=1}^{N} Expose^{marginal}(i)(G - M_i)p_B \tag{10}$$

Corollary 1 *The sequential mechanism with fixed commission is dominated by the sequential mechanism with dynamic commission, as far as the platform's expected profit is concerned.*

The proof is straightforward, joining Proposition 3 and Theorem 1.

The improvement achieved with dynamic commission structure compared to fixed commission structure is highly affected by the network structure, in particular the connectivity level (i.e., the number of partners each follower is connected to). In general as the connectivity increases the platform can reach a greater number of unique followers for each given number of promoting partners. With the core mechanism, this is the sole influence over expected profit. This is also the case with the sequential mechanism whenever using dynamic commission, as the platform is taking over the entire partners' expected profit. With the fixed-commission sequential mechanism, there is an additional influence associated with the increase in network connectivity—the increased connectivity results in a greater disparity in individual partner, favoring those that are first in the sequence. This latter phenomena has a negative effect, as when the commission is fixed, in order to incentivize partner i to promote, the offered (fixed) commission needs to be increased, resulting in a decreased expected profit for the platform.

To illustrate, we provide two different scenarios in which connectivity increases in the form of increasing the maximum number of shared partners each follower is associated with (keeping the actual number of shared connections uniform between 1 and the maximum number). The two scenarios differ in the assumption used regarding the total population size. In the first, we keep the number of followers per partner constant, hence an increase in connectivity leads to a decrease in the total population size. In the second scenario, we keep the total size of the population constant, hence an increase in connectivity requires an increase in the number of followers per partner. Both scenario use the parameters $(N = 50, k = *, M = *, c = 10, G = 15, p_B = 0.1, p_W(w) = 1/N_{max} \forall w \in 1...N_{max})$, where N_{max} is the maximum number of partners to whom a follower can be connected. For the first scenario we use $k = 40$ whereas for the second we set the population size to 350 and calculate k accordingly. The commission used is the expected-profit-maximizing one.

For both scenarios we analyze the two variants of the sequential mechanism (fixed versus dynamic commission). For the first scenario we introduce Figure 13 which depicts the platform's expected profit as a function of the maximum number of partners to whom a follower can be connected (keeping the number of followers per partner in the network fixed).

From the figure we observe that indeed as given in Corollary 1, the resulting SPNE with a fixed commission is dominated by the resulting SPNE with the dynamic commission structure. The general behavior of both curves reflect a decrease in the platform's expected profit as the number of shared partners a follower has increases. This results from the fact that partners become more reluctant to promote as with the increase in network density, the percentage of their followers that will first receive the promotion from them decreases. With the fixed commission, there is an additional effect taking place - the increase in the number of shared partners a follower has, results in a greater disparity in the profit of the different partners. This is illustrated in right-hand-side graph which depicts the portion (in percentages) of each partner's profit out of the total partners' profit as a function of its position in the sequence. As can be seen from the right graph, the denser the network, the greater is the inequality in individual profit—the platform needs to pay substantially more in order to incentivize any additional partner-promotion. Therefore the platform

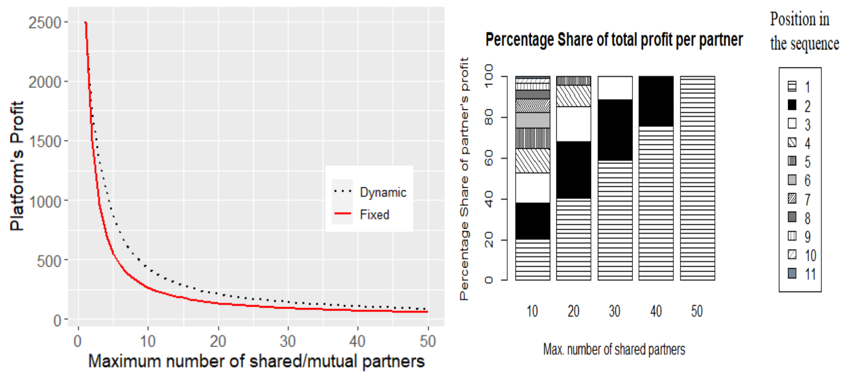


Fig. 13: Platform's expected profit when using the sequential mechanism with dynamic vs. fixed commissions structure (left) and the distribution of partner's profits according to their position in the sequence when using fixed commission structure (right) as a function of the network connectivity (reflected by the maximum number of shared partners a follower has).

ends up using a commission that incentivizes a smaller number of partners to promote, ending up in a smaller expected profit. The difference between the expected profit with the two mechanisms first increases and then decreases as connectivity increases. The increase is explained by the decrease in the number of unique followers reached with each additional promoting partner and the increase in the profit partners secure for themselves when promoting, as the connectivity increases. The decrease results from the decrease in the number of partners that eventually promote with both schemes, as connectivity increases.

An opposite trend is illustrated in Figure 14, which uses the second scenario specified above, i.e., differs only in the assumption made regarding the population size (keeping it fixed and only makes the different adaptations in the number of followers assigned to each partner). Here, the profit increases as the connectivity increases. This is attributed to the ability to reach more unique followers with each promotion, due to the increase in the number of followers each partner has. Regardless of the behavior of the expected profit, we observe here also that, as given in Corollary 1, the fixed-commission-based mechanism is dominated by the dynamic one.

While Theorem 1 proves that the sequential mechanism guarantees the maximum expected profit that can be obtained with pure-strategy equilibria in the core mechanism, the choice of using the former will depend on whether or not it also dominates mixed-strategy equilibria of the latter. Theorem 2 proves that indeed such domination holds.

Theorem 2 *Any mixed strategy equilibrium of the core mechanism is strictly dominated (as far as the platform's expected profit is concerned) by the SPNE of the sequential mechanism with dynamic commission.*

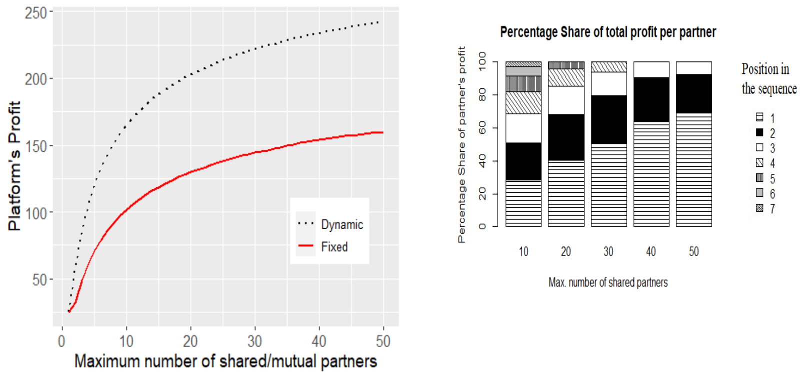


Fig. 14: Platform's expected profit when using the sequential mechanism with dynamic vs. fixed commissions structure (left) and the distribution of partner's profits according to their position in the sequence when using fixed commission structure (right) as a function of the network connectivity (reflected by the maximum number of shared partners a follower has).

Proof. A mixed equilibrium results in an induced distribution over the number of promoting partners, as captured in (5). The expected overall cost incurred by the partners when $i \leq N$ of them choose to promote is $c \cdot i$ and consequently the expected overall cost given the induced distribution is $\sum_{i=0}^{N} \binom{N}{i} p^i (1-p)^{N-i} c \cdot i$. Recall that the expected profit of any partner P_i which mixes between promoting and not promoting is zero. Meaning that the expected commission payment made by the platform, according to (5), is equal to the overall expected cost for the partners, formally: $\sum_{i=0}^{N} \binom{N}{i} p^i (1-p)^{N-i} c \cdot i = \sum_{i=0}^{N} \binom{N}{i} p^i (1-p)^{N-i} Expose(i) \cdot Mp_B$. Therefore the platforms' expected profit can be expressed as:

$$B_{core}^{platform}(p) = \sum_{i=0}^{N} \binom{N}{i} p^i (1-p)^{N-i} (Expose(i) Gp_B - c \cdot i). \qquad (11)$$

Now consider the expected profit with the sequential mechanism with dynamic commission. Here as well the platform fully covers the promoting partners' expected cost, which equals $c \cdot i$, and its expected gain is $Expose(i) Gp_B$. However, unlike with the mixed equilibrium case, here the platform gets to explicitly choose the number of partners i that will promote, through the structure of the commission it offers. Formally, it chooses an i that maximizes $Expose(i) Gp_B - c \cdot i$.

Consider a random variable $X = Expose(i) Gp_B - c \cdot i$ where $0 \leq i \leq N$ which follows a binomial distribution with parameters N and p. Since the expected value of a binomial random variable is generally less the maximum value assumed by the random variable, finally, we obtain that $max_i (Expose(i) Gp_B - c \cdot i) > \sum_{i=0}^{N} \binom{N}{i} p^i (1-p)^{N-i} (Expose(i) Gp_B - c \cdot i)$, i.e., the expected profit with the sequential mechanism is greater than with the mixed strategy equilibrium of the core mechanism. □

Notice that the above proof also holds for the case where a non-symmetric mixed-strategy equilibrium is used in the core mechanism, and even for the case where some of the partners use pure strategy and some mix. In both cases we still get a distribution over the number of participating partners. The only difference is that those partners using a pure strategy in the core mechanism are actually making a positive profit (i.e., requiring a greater payment from the platform's side). Everything else in the proof remains the same.

Theorem 2 has two important implications. First, since according to Theorem 1 the SPNE of the sequential mechanism provides the same expected profit as the best pure-strategy Nash equilibrium of the core mechanism, and as the former dominates any mixed-strategy equilibrium that holds in the core mechanism, then when having full control over the commission offered M, the platform will always prefer a pure-strategy Nash equilibrium over any mixed-strategy equilibrium. This is illustrated in Figure 15 which depicts the profit of the core mechanism with the pure and mixed strategies as a function of the cost of promotion c, for a setting $(N = 50, k = 25, M^*, c = *, G = 15, p_B = 0.1, p_W(w) = 1/N\, \forall w \in 1...N)$ when the profit-maximizing commission M^* is used with each of the mechanisms. From the figure we observe that indeed the pure-strategy equilibrium yields a higher expected profit compared to the mixed-strategy one.

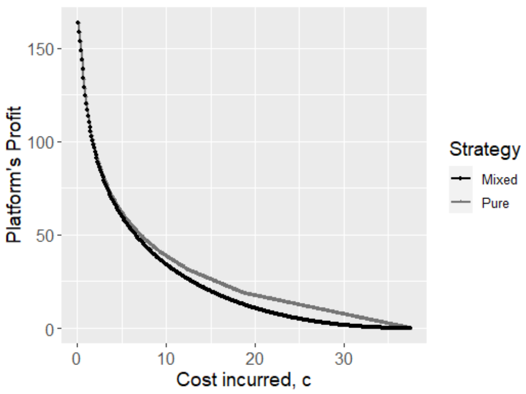


Fig. 15: The platform's expected profit under different equilibria, for different promotion cost c values, when using the profit-maximizing commission.

The difference between the expected profit with the pure-strategy and mixed-strategy equilibria is small with relatively high and low promotion costs. This is attributed to the induced probability distribution over the number of promoting partners. With low and high costs, the variance is relatively small, as the distribution is highly skewed. Therefore the expected number of promoting partners in mixed-strategy equilibrium is quite similar to the number of promoting partners in pure-strategy equilibrium. For other costs, the variance is quite substantial, resulting in performance degradation compared to the pure-strategy case.

Also, the pure-strategy equilibrium with the highest profit has the same number of promoting partners as that of the sequential mechanism with a dynamic commission structure which leads us to the second implication of Theorem 2 which is that the platform can guarantee highest profit of the pure-strategy Nash equilibrium simply by switching to the sequential mechanism with the dynamic commissions. We note that even in cases where the platform is forced to use the core mechanism, the sequential mechanism can be used to facilitate the calculation of the M value that will maximize the expected profit of the platform if the pure-strategy equilibrium is to be used. Simply solve (3) taking i to be the integer part of the solution to $\frac{c}{Expose^{marginal}(i)p_B} = G$. The latter i value is the number of promoting partners according to the sequential mechanism with dynamic commission. Therefore solving (3) with that i will guarantee the use of the pure-strategy equilibrium that results in the same expected profit as the one achieved with the latter mechanism, and according to Theorem 1 it is the expected-profit maximizing equilibrium.

5 Discussion and Conclusions

With tech giants like Twitter realizing the ramifications of small changes in mechanism design [29] and as affiliate marketing is becoming an established key source of online income for hundreds of thousands over the internet, the choice of the mechanism according to which it should be managed becomes more acute. The importance of the equilibrium analysis provided in this paper is in being the first, to the best of our knowledge, to model the participants of affiliate marketing as self-interested agents and to consider the strategic choices of partners in affiliate marketing while taking into account the complete set of influencing factors that hold in real-life. These include the information disclosed by the platform, the promotion costs and the possible overlap between the followers, as is the case of social networks. In fact, this is the first attempt to study information design in the context of affiliate marketing.

The analysis is split into three main parts, namely, the core mechanism, the sequential mechanism and a dynamic pricing extension to the sequential mechanism. The core mechanism models affiliate marketing as it is done today, with a pre-determined fixed commission offered to the partners and with no other information revealed to them. The sequential mechanism proposes a minor change in information disclosure where the number of times a given listing is viewed is revealed to every partner considering the opportunity. Finally, the dynamic pricing extension to the sequential mechanism proposes a pricing mechanism where the commission offered to the partner is based on her position in the sequence.

The theoretical analysis is complemented by various numerical investigations revealing much insight related to the effect of the network structure (number of followers and distribution of shared followers) as well as various other model parameters (the promotion cost and offered commission) over the platform's expected profit.

The proposed mechanism, according to which information about the number of times an opportunity has been reviewed is disclosed to the partners and the commission offered is dynamic, is both easy to implement and encapsulates several important inherent advantages. The primary advantage is of course the guarantee to obtain an expected profit that equals the one obtained with the most profitable equilibrium among those that hold in the core mechanism in a multi-equilibria scenario. Others relate to computational aspects and the nature of the equilibrium (SPNE vs. NE). Furthermore, the sequential mechanism does not require the platform to determine an order in which partners review opportunities. Instead, they are serviced based on the order of arrival, hence no fairness issues arise.

We note that an alternative way for enforcing the expected-profit maximizing pure-strategy equilibrium in the core mechanism is to simply limit the number of partners who can promote (and set the commission accordingly, such that all promoting partners end up with no profit), as discussed in the former section. Yet, limiting the number of partners that can promote a given product is probably less appealing for the platform compared to simply providing information on how many have already reviewed the opportunity, as it may make partners reluctant to subscribe to the platform (which is bad, especially when some subscription fee is charged, as in some of the platforms).

Indeed, much like in many other game-theoretic analyses of markets, our model relies on the assumption that partners are a priori homogeneous. The extension to the heterogeneous case is mostly technical and does not add much in terms of insights. The extraction of the equilibrium is quite the same, with the only change required being the assignment of a different promotion probability to different partners and some basic adaptation in the way the number of followers being exposed to the promotion is being calculated. With heterogeneous partners, however, two new challenges arise. The first is computational one, as equilibrium calculation becomes combinatorial. The second is the ordering used in the sequential mechanism. While this latter factor has no influence in the a priori homogeneous case, it has a great influence once the partners are heterogeneous. Consequently, we can quite easily construct settings where the sequential mechanism is inferior to the core mechanism and vice-versa.

We see many directions for extending this work. One natural direction is the analysis of multi-platform competition. This may become especially important as many platforms nowadays are becoming exclusive (due to contracts with manufacturers). A tangential direction for future research is the analysis of affiliate marketing where the cost of promotion c varies as a probability distribution in a cost range. Another interesting direction for future empirical research —correlating the theoretical results with empirical findings related to the commissions used, the effort associated with promoting and the structure of the social network in real-world settings. With affiliate marketing being a largely internet-based marketing strategy, we also hope that our analysis sparks interest on research on affiliate marketing in the domain of multi-agent systems where automated agents help the platforms refine their commission structure and reap higher profits while similar automated agents can help the partners take more informed decisions on whether or not to promote the product.

Acknowledgements Preliminary results of this research appear in Proceedings of the First International Conference on Distributed Artificial Intelligence (DAI'19). This research has been partly supported by the ISRAEL SCIENCE FOUNDATION (grants No. 1162/17 and 1958/20) and the EU project TAILOR under Grant 992215.

References

1. Akçura, M.T.: Affiliated marketing. Information Systems and e-Business Management **8**(4), 379–394 (2010)
2. Amarasekara, B.R., Mathrani, A.: Controlling risks and fraud in affiliate marketing: A simulation and testing environment. In: 2016 14th Annual Conference on Privacy, Security and Trust (PST), pp. 353–360. IEEE (2016)
3. Audrezet, A., de Kerviler, G., Moulard, J.G.: Authenticity under threat: When social media influencers need to go beyond self-presentation. Journal of business research **117**, 557–569 (2020)
4. Avagyan, V.: Marketing strategies for enhancing word of mouth: sales acceleration and affiliate programs. Ph.D. thesis, Universidad Carlos III de Madrid (2012)
5. Ballestar, M.T., Grau-Carles, P., Sainz, J.: Customer segmentation in e-commerce: Applications to the cashback business model. Journal of Business Research **88**, 407–414 (2018)
6. Benediktova, B., Nevosad, L.: Affiliate marketing: Perspective of content providers. Master's thesis, Department of Business Administration and Social Sciences, Lulea University of Technology (2008)
7. Bhattacharya, S., Roy, A., Kumar, S., Wattal, S.: Search engine advertising strategies when contracting with an online search infomediary: A game-theoretic model
8. Chachra, N., Savage, S., Voelker, G.M.: Affiliate crookies: Characterizing affiliate marketing abuse. In: Proceedings of the 2015 Internet Measurement Conference, pp. 41–47. ACM (2015)
9. Cheng, P., Chen, L., Ye, J.: Cooperation-aware task assignment in spatial crowdsourcing. In: 2019 IEEE 35th International Conference on Data Engineering (ICDE), pp. 1442–1453. IEEE (2019)
10. Cheng, P., Xin, H., Chen, L.: Utility-aware ridesharing on road networks. In: Proceedings of the 2017 ACM International Conference on Management of Data, SIGMOD '17, p. 1197–1210. Association for Computing Machinery, New York, NY, USA (2017). DOI 10.1145/3035918.3064008. URL https://doi.org/10.1145/3035918.3064008
11. Daniele, R., Frew, A.J., Varini, K., Magakian, A.: Affiliate marketing in travel and tourism. Information and Communication Technologies in Tourism 2009 pp. 343–354 (2009)
12. Dayama, P., Karnik, A., Narahari, Y.: Optimal incentive timing strategies for product marketing on social networks. In: Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems-Volume 2, pp. 703–710. International Foundation for Autonomous Agents and Multiagent Systems (2012)
13. Devari, A., Nikolaev, A.G., He, Q.: Crowdsourcing the last mile delivery of online orders by exploiting the social networks of retail store customers. Transportation Research Part E: Logistics and Transportation Review **105**, 105–122 (2017)

14. Doles, B.: How to capitalise on the gig economy (2019). URL https://www.tune.com/blog/how-to-capitalize-on-the-gig-economy/
15. Duffy, D.L.: Affiliate marketing and its impact on e-commerce. Journal of Consumer Marketing **22**(3), 161–163 (2005)
16. Dwivedi, Y.K., Rana, N.P., Alryalat, M.A.A.: Affiliate marketing: An overview and analysis of emerging literature. The Marketing Review **17**(1), 33–50 (2017)
17. Edelman, B., Brandi, W.: Risk, information, and incentives in online affiliate marketing. Journal of Marketing Research **52**(1), 1–12 (2015)
18. Emek, Y., Karidi, R., Tennenholtz, M., Zohar, A.: Mechanisms for multi-level marketing. In: Proceedings of the 12th ACM conference on Electronic commerce, pp. 209–218. ACM (2011)
19. Fagan, C.: How the financial diet actually makes money (2017). URL https://thefinancialdiet.com/how-the-financial-diet-actually-makes-money/
20. Fainmesser, I.P., Galeotti, A.: The market for online influence. Johns Hopkins Carey Business School Research Paper (18-13) (2019)
21. Fainmesser, I.P., Galeotti, A.: Pricing network effects: Competition. Johns Hopkins Carey Business School Research Paper (17-08) (2019)
22. Fiore, F., Collins, S., Foreword By-Marciano, J.L.: Successful affiliate marketing for merchants. Que Corp. (2001)
23. Garnefeld, I., Eggert, A., Helm, S.V., Tax, S.S.: Growing existing customers' revenue streams through customer referral programs. Journal of Marketing **77**(4), 17–32 (2013)
24. Guo, Z.: Optimal decision making for online referral marketing. Decision Support Systems **52**(2), 373–383 (2012)
25. Ho, Y.C.C., Ho, Y.J.I., Tan, Y.: Online cashback pricing: a new affiliate strategy for e-business. In: Proceedings of the 34th international conference on information systems (ICIS) (2013)
26. Hoffman, D., Novak, T.: How to acquire customers on the web? Harvard Business Review **78**, 179–88, 218 (2000)
27. Huws, U., Spencer, N., Joyce, S.: Crowd work in europe: Preliminary results from a survey in the uk, sweden, germany, austria and the netherlands (2016)
28. Ipeirotis, P.G., Provost, F., Wang, J.: Quality management on amazon mechanical turk. In: Proceedings of the ACM SIGKDD workshop on human computation, pp. 64–67 (2010)
29. Jackson, L., Ibekwe, D.: Jack dorsey on twitter's mistakes (2020). URL https://www.nytimes.com/2020/08/07/podcasts/the-daily/Jack-dorsey-twitter-trump.html
30. Kempe, D., Kleinberg, J., Tardos, É.: Maximizing the spread of influence through a social network. In: Proceedings of the ninth ACM SIGKDD international conference on Knowledge discovery and data mining, pp. 137–146. ACM (2003)
31. Lee, K., Webb, S., Ge, H.: The dark side of micro-task marketplaces: Characterizing fiverr and automatically detecting crowdturfing. In: Proceedings of the International AAAI Conference on Web and Social Media, vol. 8 (2014)
32. Li, B., Hao, D., Zhao, D., Zhou, T.: Mechanism design in social networks. In: Proceedings of the Thirty-First AAAI Conference on Artificial Intelligence, pp. 586–592 (2017)
33. Libai, B., Biyalogorsky, E., Gerstner, E.: Setting referral fees in affiliate marketing. Journal of Service Research **5**(4), 303–315 (2003)
34. Liu, Y., Guo, B., Du, H., Yu, Z., Zhang, D., Chen, C.: Poster: Foodnet: Optimized on demand take-out food delivery using spatial crowdsourcing. In: Proceedings of the 23rd Annual International Conference on Mobile Computing and Networking, pp. 564–566 (2017)
35. Lobel, I., Sadler, E., Varshney, L.R.: Customer referral incentives and social media. Management Science **63**(10), 3514–3529 (2016)
36. Ma, M.: Performance-based marketing on the internet. AMCIS 2006 Proceedings p. 225 (2006)
37. Maile, F.L.: Artificial intelligence and big data in affiliate marketing: A deep dive into the tools, techniques, and opportunities. Master's thesis, University of Applied Sciences Aalen (2018)

38. Malik, Z.K., Fyfe, C., Crowe, M.: Priority recommendation system in an affiliate network. Journal of Emerging Technologies in Web Intelligence **5**(3), 222–229 (2013)
39. Mariussen, A., Daniele, R., Bowie, D.: Unintended consequences in the evolution of affiliate marketing networks: a complexity approach. The Service Industries Journal **30**(10), 1707–1722 (2010)
40. Mathur, A., Narayanan, A., Chetty, M.: Endorsements on social media: An empirical study of affiliate marketing disclosures on youtube and pinterest. Proceedings of the ACM on Human-Computer Interaction **2**(CSCW), 1–26 (2018)
41. Mazurek, G., Kucia, M.: Potential of affiliate marketing. In: International Conference Management of Technological Changes, vol. 7, pp. 1–4 (2011)
42. Mizuno, M.: How consumer-generated advertising works: An empirical agent-based simulation. In: Advances in computational social science, pp. 17–29. Springer (2014)
43. Mordkovich, B., Mordkovich, E.: Pay-per-click search engine marketing handbook: low cost strategies to attracting new customers using Google, Yahoo & other search engines. Lulu. com (2005)
44. Morris, N.: Understanding digital marketing: Marketing strategies for engaging the digital generation. Journal of Direct, Data and Digital Marketing Practice **10**(4), 384–387 (2009). DOI 10.1057/dddmp.2009.7. URL https://doi.org/10.1057/dddmp.2009.7
45. Nash, J.: Non-cooperative games. Annals of mathematics pp. 286–295 (1951)
46. Olbrich, R., Schultz, C.D., Bormann, P.M.: The effect of social media and advertising activities on affiliate marketing. International Journal of Internet Marketing and Advertising **13**(1), 47–72 (2019)
47. Papatla, P., Bhatnagar, A.: Choosing the right mix of on-line affiliates: How do you select the best? Journal of Advertising **31**(3), 69–81 (2002)
48. Patrick, Z., Hee, O.C.: Factors influencing the intention to use affiliate marketing: A conceptual analysis. International Journal of Academic Research in Business and Social Sciences **9**(2) (2019)
49. Pei, A., Mayzlin, D.: Influencing the influencers. Available at SSRN 3376904 (2019)
50. Ryan, D.: Understanding Digital Marketing: Marketing Strategies for Engaging the Digital Generation. Kogan Page Publishers (2016)
51. Schmitt, P., Skiera, B., den Bulte, C.V.: Referral programs and customer value. Journal of Marketing **75**(1), 46–59 (2011). DOI 10.1509/jm.75.1.46. URL https://doi.org/10.1509/jm.75.1.46
52. Schomer, A.: E-commerce in digital media, business insider intelligence report (2018)
53. Sen, I., Aggarwal, A., Mian, S., Singh, S., Kumaraguru, P., Datta, A.: Worth its weight in likes: Towards detecting fake likes on instagram. In: Proceedings of the 10th ACM Conference on Web Science, WebSci '18, p. 205–209. Association for Computing Machinery, New York, NY, USA (2018). DOI 10.1145/3201064.3201105. URL https://doi.org/10.1145/3201064.3201105
54. Snyder, P., Kanich, C.: No please, after you: Detecting fraud in affiliate marketing networks. In: Proceedings of the Workshop on the Economics of Information Security (2015)
55. Tang, S.: Profit-driven team grouping in social networks. In: Thirty-First AAAI Conference on Artificial Intelligence (2017)
56. Taylor, M., Marsh, G., Nicol, D., Broadbent, P.: Good work: The taylor review of modern working practices (2017)
57. Wolves, F.: 14 gig economy jobs to make money (2020). URL https://financialwolves.com/gig-economy-jobs/
58. Wood, A.J., Graham, M., Lehdonvirta, V., Hjorth, I.: Good gig, bad gig: autonomy and algorithmic control in the global gig economy. Work, Employment and Society **33**(1), 56–75 (2019)
59. Zenonos, S., Tsirtsis, A., Tsapatsoulis, N.: Twitter influencers or cheated buyers? In: 2018 IEEE 16th Intl Conf on Dependable, Autonomic and Secure Computing, 16th Intl Conf on Pervasive Intelligence and Computing, 4th Intl Conf on Big Data Intelligence and Computing and Cyber Science and Technology Congress(DASC/PiCom/DataCom/CyberSciTech), pp. 236–242 (2018)
60. Zhao, D., Li, B., Xu, J., Hao, D., Jennings, N.R.: Selling multiple items via social networks. In: Proceedings of the 17th International Conference on Autonomous Agents and MultiAgent Systems, pp. 68–76 (2018)

61. Zhou, Y.W., Cao, B., Tang, Q., Zhou, W.: Pricing and rebate strategies for an e-shop with a cashback website. European Journal of Operational Research **262**(1), 108–122 (2017)