Agent's Multiple Inquiries for Enhancing the Partnership Formation Process*

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Abstract. The concept of sequential two-sided search is widely used in partnerships formation applications in various domains. However, when considering the implementation of the method in Multi-Agent Systems (MAS), one may notice a strong incentive for agents to deviate from the traditional equilibrium sequential search strategy towards an extended search method which combines simultaneous inquiries in each search round. In the current paper we explore such a model, where agents of a specific type can use this kind of simultaneous search technique. Since all agents types strategies take into consideration the other agents' strategies, the main focus is on the equilibrium analysis. By introducing the agents' expected utility functions, we manage to present a complete equilibrium based analysis for the new model combining the simultaneous inquiries technique. The specific characteristics of the equilibria, derived from the analysis, allow us to suggest efficient algorithms for calculating each agent's strategy. As a complementary application for the proposed model, we suggest the buyer-seller two-sided search process in C2C eMarketplace environments. Here, buyer agents utilize the new search technique in order to enforce a new equilibrium which yields a better utility for themselves. The perceived improvement in the agents performance in comparison to the traditional two-sided search method is demonstrated through simulations.

1 Introduction

The concept of two-sided search for forming partnerships among agents can be found in many MAS applications [9]. The key issue for each agent engaged in such search process is to determine the set of agents it is willing to form a partnership with. The number of possible partners the agents seek is application dependent. In this paper we focus on partnerships where each agent is satisfied with only one partner. Typical applications that make use of size-two partnership formation processes include buyer-seller, peer-to-peer media exchange, dual long distance call partnering termination-services [15], dual backup services [14] etc. The main characteristic of these applications is that an agent can gain a utility only if it eventually partners with another agent. However, once a partnership is formed,

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adding additional agents as partners does not produce any additional benefit. The search process for such partnerships is considered two-sided as all agents in the environment engage in search. Thus a partnership eventually formed is the result of the combined search activities of the agents forming it.

Traditionally, applications where individuals were engaged in dual search activities (e.g. marriage and labor markets [2]), considered pure sequential search where each party samples and evaluates one potential partner at a time. Nevertheless, while considering the migration of such models into MAS environments, one must also take into consideration an important strength of autonomous agents (in comparison to humans) - their ability to handle an enormous amount of information and to maintain interaction with several other agents in parallel [5]. This research is motivated by such strength, i.e., allowing agents to simultaneously interact with several other agents during each search round. We aim to show that the use of the new method derives different equilibrium strategies (in comparison to the pure sequential model) for the agents. The results of the analysis suggests that as the choice of using the method is given to one of the agents types, this type can only improve and never worsen its expected utility.

In order to demonstrate the suggested methodology, we use the legacy two-sided agents search application - the buyer-seller search in Electronic commerce. More specifically, we use the C2C (Consumer-to-Consumer) segment, where consumers can be found at both ends of the transaction. We consider a C2C marketplace with numerous buyer and seller type agents. Each agent explores the market for opportunities to buy or sell a specific item, equipped with its owner's personal preferences and requirements. Once a buyer agent and a seller agent become acquainted, they interact, following given conventions defined by ontology and language. If the buyer agent's preferences and requirements for the product attributes and functionalities are met, then a possible transaction between the two of them can be considered associated with specific terms (including the price) and policies. Dual commitment to the proposed transaction will result in an agreement and an immediate utility for both sides. Otherwise both agents will continue their search, looking for other candidates. Notice that the seller agents in C2C marketplaces usually have a single item (or a limited quantity) they wish to sell on an irregular basis. Therefore, even though the seller agents do not proactively search for buyer agents, they are active in a selective manner in order to maximize their total utility.

The utility an agent gains from any given transaction, is a function of many factors. While for simple products (like CDs), it is mostly a matter of price, for more complex products, the purchasing decision generally requires a complex trade-off between a set of preferences. We apply the multi-attribute utility theory (MAUT) [8], for analyzing preferences with multiple attributes in our agent-based trading mechanism [9]. The reason for using such a function is two-folded. First, most C2C users buy and sell assorted items that are often difficult to describe, and are not easily evaluated. Since in most cases it will be a used item, the value for the buyer will be influenced mostly by the specific functionalities (including attributes like color, size, etc.), quality and current condition of the product. Second, the transaction that will eventually be made will include many terms and policies (concerning warranties, return policy, payment policy, delivery time and policy, insurance for the delivery, etc.). All these terms have value for both sides and can be critical to their buying/selling decision, regardless of the manner of shopping [5]. Adding reputation and trust factors to the agents' considerations, and keeping in mind that in many terms and policies buyers and sellers do not have directly competing interests [4], one may conclude that similar potential transactions may suggest different utilities for different buyers and sellers agents.

The general framework of our model consists of an environment populated by many agents, where each agent is associated with one of two types. Partnerships are formed between single agents of the two types. Each agent is self-interested and has no a-priori knowledge regarding the utility that can be obtained by partnering with specific agents in the environment. In order to find an appropriate partner, each of the agents interacts with other agents of the opposite type, evaluating the perceived utility from partnerships with these agents. The perceived utility from any interaction is associated with a distribution function (widely common assumption in traditional two-sided search models [2]). In the current context, we limit the possibility of using the simultaneous search for one of the types. Such a scenario is applicable in several environments, as will be discussed in the following sections. The methodology and results presented in this paper can be used as an infrastructure for future research, considering the model where all agents types can use the new technique.

An inherent part of such a dual search model is the search "cost", reflecting the resources required by the agent to perform its search activities. This includes the cost associated with the interaction between agents, locating other agents, analyzing and comparing offers, decision making, etc. Many authors have argued that advances in communication technologies reduce search costs and other market inefficiencies. However the general agreement is that these cannot be ignored completely [1]. These should be considered when computing the total expected utility for the agent, given a specific search strategy.

Upon meeting a potential partner agent, at any given stage of the search, the agent needs to decide whether to form a partnership with the agent it has met or to continue its search. If it will attempt to join the current agent and that agent will commit to the partnership as well, then it will immediately gain the expected utility from the partnership; Otherwise, the agent will need to continue searching, bearing additional search costs. In the latter case the agent's future expected utility will be derived from the benefit future interactions might offer, as well as the encountered agents' willingness to form a partnership. Therefore, an agent's decision of whether to join the current possible partner depends on the strategies set by the other agents. In a similar manner their strategies also depend on the decision of the agent. Thus, we are looking for strategies that are in equilibrium. As we will show in this paper, the equilibrium will consist of reservation value based strategies. That is, the agent will set a reservation value, x, accepting a partnership yielding a utility greater than or equal to x and rejecting all partnerships with agents that yield a utility lower than x.[3]

An important output of our work is the in-depth analysis we suggest for the model, shedding light on unique characteristics of the agents equilibrium policies. Nevertheless, our main contribution is in the integration of the simultaneous search capability into the agent's search strategy in the two-sided model. This suggests a more efficient search that significantly reduces some of the agent's fixed-natured search costs and increases its overall performance (in terms of the perceived utility). Throughout the paper we suggest efficient tools for the agent to find the optimal number of simultaneous interactions in a search round. The proposed algorithms, can also be useful for the traditional two-sided pure sequential search model as a specific case.

In the next section we address relevant multi-agent and search literature. In section 3 we present the model. An equilibrium analysis and calculation algorithms are given in section 4. We conclude and present directions for future research in section 5.

2 Related Work

The search process for partners, often associated with agent matchmaking concept, has wide evidence in literature [9, 17]. In its wider extent it can be seen as part of the multi-agent coalition formation model found in the electronic market [7, 10]. While some mechanisms assume that an agent can scan as many agents as needed, others use a central matcher or middle agents [3]. Few have considered the problem of finding matches for cooperative tasks without the help of a predefined organization or a central facilitator [13, 14]. However, to the best of our knowledge, a distributed simultaneous search for partners has not been studied.

The traditional two-sided search application initially evolved from the area of search theory ([11], and references therein). These models focused on establishing optimal strategies for the searcher, assuming no mutual search activities and were classified as one-sided search. In an effort to understand the effect of dual search activities, the "two-sided" search research followed [16, 2].

The transition of these models into MAS environments is non-trivial. While traditional models assume poisson arrival rates for new opportunities, we do see room for agents that control their search intensity and even initiate simultaneous interactions for improving their utility. The use of variable sample sizes was suggested for the one-sided search [12], however, since there is only one searcher in these models this became a simple optimization problem, with no equilibrium concerns. The equilibrium concept is the key issue in the simultaneous two-sided

[3] The reservation value of the search strategy is different than a reservation price usually associated with a buyer or a seller that are not involved in a search. While the reservation price denotes an agent's true evaluation of a specific potential transaction, the reservation value of a search strategy is mainly a lower bound for accepted transactions, derived from the expected utility optimization considerations.

search, as the agent needs to consider the effect of its strategy both on the other agents' strategies and on its own performance. This significantly increases the complexity of the problem. Our model also differs in the way search "costs" are modelled. Unlike most traditional search models, where search "costs" are modelled by the discounting of the future flow of gains, we see these costs as an actual explicit resource the agent needs to put into the search. This is mainly because the search in MAS environments will usually last a few days\hours, and will result in an immediate utility. Lastly, notice that while traditional models [2] are more concerned with describing the equilibrium equations, we also require algorithms and methods for deriving the agents policies for different settings and the distributed computation of the equilibrium strategy.

3 The Two-Sided Simultaneous Search Model

Consider an environment populated with numerous agents of two types, where each agent is interested in forming a partnership with an agent of the opposite type. For illustrative purposes, we'll continue with the model denoting the two types as buyer and seller agents, residing in a C2C marketplace environment, interested in buying or selling various items. Any random interaction between a seller agent and a buyer agent, may yield a transaction for exchanging the item according to well defined specific terms and policies. As suggested in the introduction, the perceived utilities from any suggested transaction between two specific agents, denoted by U^s for the seller agent and U^b for the buyer agent, can be seen as randomly drawn from a population with p.d.f. $f^s(U^s)$ and c.d.f. $F^s(U^s)$ for the seller agent and $f^b(U^b)$ and $F^b(U^b)$ for the buyer agent ($0 \leq U^s, U^b < \infty$). We assume that buyer (seller) agents, while ignorant of individual seller (buyer) agents' offers (preferences) are acquainted with the overall utility distributions (a common assumption in search models, see for example [16]).

We consider simultaneous interactions only by agents of a specific type. This suits the electronic marketplace applications, as in current C2C markets sellers are usually approached by buyers, and they do not approach buyers in a proactive manner. Thus, at any stage of its search each buyer agent randomly encounters N seller agents interested in selling an item similar to the one the buyer agent wishes to buy. Out of the set of N potential transactions, the buyer agent will focus on the "best opportunity", i.e., the one with the highest utility, denoted by U^b_N. This is in comparison to the traditional pure sequential model [2], where each buyer agent is acquainted with only one seller agent in a search stage.

Each agent needs to allocate resources to maintain its search. We consider this "cost" per search stage to be composed of a fixed cost and a variable cost. Setting α_b and β as the fixed and variable components of the buyer agents' search cost, respectively, we obtain a total search cost per search stage of $\alpha_b + \beta N$. The seller agents search sequentially, and thus their search costs can be seen as α_s. We assume the agent's utility from a given transaction, as well as the resources required for maintaining the search, can be measured on a similar scale. Thus the total search utility can be obtained by subtracting the search "costs" from the perceived utility for any given transaction. A model where different agents of the same type will be using different search structures may also be considered. In this case we will obtain similar sub-types of agents which can be integrated into the appropriate equations along with their distribution in the general population. In the current paper we present the analysis of the case where all agents of a specific type (e.g. buyer agents) share the same cost structure, which is applicable for most markets where the agents are supplied to the users by the market maker.

After reviewing and evaluating the potential partnership, defined by the proposed transaction, each agent will make a decision whether to commit to it. A transaction will take effect only if both agents are willing to commit to it. Otherwise both agents will resume their search according to the same cost structure. Since the agents are not limited by a decision horizon or the number of search rounds, and the interaction with other agents does not imply any new information about the market structure, their search strategy is stationary, i.e. an agent will not accept an opportunity it has rejected beforehand, and the value N will remain constant over time. As the agents are seeking to maximize their utility, they will use a reservation value based strategy.

Consider an environment where buying agents use N simultaneous interactions over each search round. The reservation values used by the buyer and seller agents will be denoted as x_N^b and x_N^s, respectively. The expected future utility of the buyer and seller agents, when using these reservation values, will be denoted as $V^b(x_N^b)$ and $V^s(x_N^s)$. Due to space considerations, from this point onward, we present only the equations associated with the buyer type agents. Unless stated otherwise, similar modifications for seller type agents can be extracted using similar methods.

After reviewing the best potential transaction, U_N^b, found over the current search round, each buyer agent has to make a decision whether to reject this opportunity and continue the search or commit to the transaction. Continuing the search will result in an expected future utility of $V^b(x_N^b)$. Committing to the potential transaction will result in a utility of U_N^b if the other agent commits as well, or otherwise it will force the agent to keep searching with an expected future total utility of $V^b(x_N^b)$. Thus the buyer agents' expected utility can be calculated as[4]:

$$\begin{aligned} V^b(x_N^b) = E\Big[&U_N^b \bullet 1[(U_N^b \geq x_N^b) \cap (U^s \geq x_N^s)] + \\ &+ V^b(x_N^b) \bullet 1[(U_N^b \leq x_N^b) \cup (U^s \leq x_N^s)] - \alpha_b - \beta N\Big] \end{aligned} \tag{1}$$

Here, $\bullet 1[(U_N^b \geq x_N^b) \cap (U^s \geq x_N^s)]$ represents the indicator of the event where a specific buyer agent and its "best" encountered seller agent (in the current search round) found the perceived utility from a transaction between the two of them to be greater or equal to their reservation values $((U_N^b \geq x_N^b) \cap (U^s \geq x_N^s))$.

[4] Detailed formulation and proofs, as well as the seller agents modifications, can be found at http://www.cs.biu.ac.il/ sarit/Articles/multiSearch.pdf.

Denoting the c.d.f., p.d.f. and the mean of the maximum utility for the buyer agent in an N-size sample of sellers as F_N^b, f_N^b and $E[U_N^b]$, we attain:

$$V^b(x_N^b) = \frac{(1 - F^s(x_N^s)) \int_{y=x_N^b}^{\infty} y f_N^b(y) dy - a_b - \beta N}{(1 - F_N^b(x_N^b))(1 - F^s(x_N^s))} \tag{2}$$

The above equation can be used by each agent to calculate its expected utility, when using different reservation value strategies, given the cost search parameters and the strategy used by agents of the opposite type. From this equation we can derive an agent's reaction to changes in the other agents strategies, towards a complete equilibrium analysis. Notice that equation (2), as well as the rest of the following suggested analysis, is also applicable for the traditional pure sequential two-sided search, simply by using N=1.

4 Equilibrium Strategies

Our goal is to supply the buyer agents with tools for calculating their optimal number of simultaneous interactions, N^*, to be used in their search. For this purpose, we first analyze the equilibrium strategies that will be used by each agent, for any given number of simultaneous interactions, N. This is achieved by understanding how an agent's strategy is affected by changes in the strategy used by agents of the other type. Then, based on the analysis given, we are able to suggest an efficient algorithm to find the optimal N.

4.1 An Agent's Expected Utility Analysis

Notice that an immediate result from (2) is:

$$\lim_{x_N^b \to \infty} V^b(x_N^b) = -\infty \quad ; \quad \lim_{x_N^b \to 0} V^b(x_N^b) = E[U_N^b] - \frac{a_b + \beta N}{1 - F^s(x_N^s)} \tag{3}$$

The content of (3) is intuitive: if the reservation value x_N^b, is very large, the chances of obtaining a utility greater than this reservation value, from a given search round, are small. Thus, repeated search rounds must be taken, leading to an overall low utility. If, on the other hand, the reservation value, x_N^b, is very small, almost surely a potential transaction suggesting a better utility can be obtained during the first search round.

The following Theorems 1-3, suggests several additional important properties of the agents' expected utility function, to be used later, for designing the calculation algorithms for the agents' strategies.

Theorem 1. *The expected utility function $V^b(x_N^b)$ is quasi concave, with a unique maxima satisfying:*

$$V^b(x_N^b) = x_N^b \tag{4}$$

Sketch of Proof: Deriving equation (2) we obtain:

$$\frac{dV^b(x_N^b)}{dx_N^b} = \frac{f_N^b(x_N^b)(V^b(x_N^b) - x_N^b)}{(1 - F_N^b(x_N^b))} \equiv r(x_N^b)(V^b(x_N^b) - x_N^b) \tag{5}$$

A solution for (5) requires that $V^b(x_N^b) = x_N^b$. Note that $f_N^b(x_N^b) > 0$ implies $r(x_N^b) > 0$, hence for x_N^b satisfying $V^b(x_N^b) = x_N^b$:

$$\frac{d^2V^b(x_N^b)}{{dx_N^b}^2} = r'(x_N^b)(V^b(x_N^b) - x_N^b) + r(x_N^b)(V^{b'}(x_N^b) - 1) < 0 \tag{6}$$

Thus $V^b(x_N^b)$ and $V^s(x_N^s)$ are quasi concave with a unique maxima.□

Equality (4) is very common in models integrating a reservation value. It suggests that the expected utility when using the optimal reservation value equals the optimal reservation value. Intuitively, we can say the agent's optimal reservation value can be found when it is indifferent between the utility that can be obtained from a transaction and the utility associated with continuing the search.

Figure 1, illustrates the agents' expected utility as a function of the reservation value in two settings. The environment used in this figure contains numerous buyer and seller agents, where each interaction between any buyer and any seller agents produce utilities drawn from a triangular distribution function[5], defined over the interval (0,100). Buyer agents are associated with fixed and variable costs coefficients $\alpha_b = 2$ and $\beta = 0.5$, and seller agents are associated with a search cost $\alpha_s = 2.5$ (thus when buyer agents use N=1, all agents' search cost structures are symmetric).

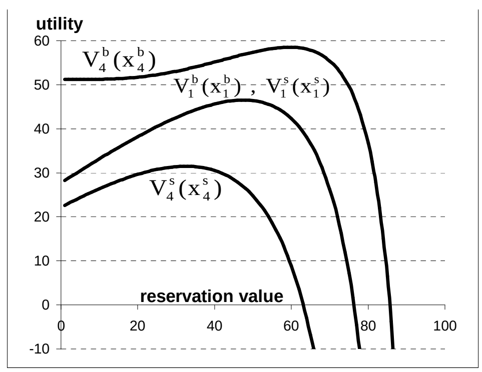

Fig. 1. Agent's expected utility function - specific environment

In the first setting, all agents use pure sequential search ($N = 1$). The middle curve describes the expected utility of any of the agents in this scenario as a function of the reservation value used (the horizontal axis). In the second setting, buyer type agents use the new simultaneous search method ($N = 4$). In this scenario buyer type agents have the incentive to use the new technique since their utility increases for any reservation value when using it (represented by the upper curve). Similarly, the expected utility for seller type agents (represented by the lower curve) always decreases when the buyer agents adopt the method.

[5] This can be related to most electronic marketplaces. It reflects a high probability to draw an opportunity producing a low utility, and vice versa.

The incentive for the buyer agents to use the combined simultaneous search technique is strong. Any single buyer agent will prefer to use more than a single interaction during a search round, if he finds the expected utility to be higher in this manner. Figure 2 demonstrates this phenomena for the uniform distribution function. As the utility varies from 0 to 1, the bottom triangular area represents all plausible α_b and β combinations where the agents will consider a pure sequential search (e.g. where the expected utility for the agents in a pure sequential equilibrium strategy is positive). Out of this area, we have isolated (on the left side) all combinations of α_b and β (setting $\alpha_s = \alpha_b + \beta$) where an agent can increase its expected utility by deviating from such a pure sequential strategy (assuming all other agents' strategies are sequential). We learn from the graph that buyer agents have an incentive to deviate from the traditional pure sequential search strategy for many plausible combinations of α_b and β values. Furthermore, the advantage of the new technique is mostly in combinations of small α_b and β values (in comparison to the average utility from a partnership), which characterizes most MAS applications.

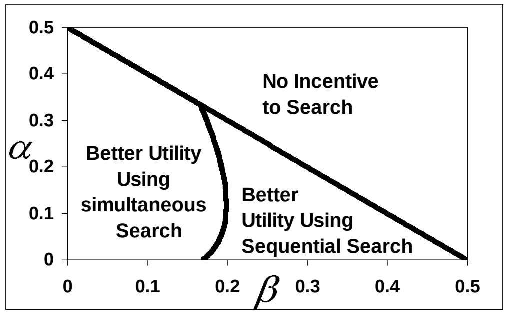

Fig. 2. Incentive for simultaneous search.

Theorem 2. *Given the reservation value that was set by the seller agents, x_N^s, the buyers agents' optimal reservation value, x_N^b satisfies:*

$$a_b + \beta N = (1 - F^s(x_N^s))(E[U_N^b] - \int_{y=0}^{x_N^b} (1 - F_N^b(y))dy) \tag{7}$$

Sketch of Proof: Deriving the expected utility given in equation (2), setting it to zero, and using integration by parts for calculating $\int_{y=x_N^b}^{\infty} y f_N^b(y)dy$, we finally obtain equation (7).□

From Theorem 2 we can conclude that the buyer agents' optimal reservation value (and thus the total utility for these agents, based on Theorem 1) decreases in α_b and β. This also has an intuitive explanation - as the search costs increase, the agent becomes less selective, reducing its reservation value. Secondly, we can conclude from Theorem 2 that the buyer agents' optimal reservation value (and thus the total utility for the agent), given the seller agent's reservation value, decreases as x_N^s increases. Similar results can be obtained for the seller.

Both equation (4) and (7), and their appropriate modifications for the seller agent, can be used for calculating the optimal reservation values of any agent type in the search, given the reservation values used by the agents of the opposite type. However, for most distribution functions, it is impossible to extract x_N^s and x_N^b using direct calculations. Fortunately, the characteristics of the optimal strategies (given the other agents' reservation values) as proved in Theorems 1-2 enable us to suggest an efficient algorithm for estimating these values up to any required precision level.

The idea is that using equation (2) for calculating the expected utility $V^b(x_N^b)$, and comparing the result with x_N^b, we can clearly determine if the current reservation value used is greater or lesser than the optimal value. As long as $V^b(x_N^b) > x_N^b$ holds, the reservation value used is smaller than the optimal reservation value and vice versa. Bounding the interval in which the optimal reservation value resides, we can use a binary search for finding a good estimation of the value. Notice that in most cases the distribution functions of U^s and U^b are finite (assuming a person's utility from a specific exchange is finite). However, we can find a bounding interval even for an infinite distribution function as Theorem 3 below suggests.

Theorem 3. *The values x_{N+}^b and x_{N-}^b satisfying $(1-F^s(x_N^s))\int_{y=x_{N+}^b}^{\infty} y f_N^b(y)dy = \alpha_b + \beta N$ and $x_{N-}^b = E(U_N^b) - \frac{\alpha_b}{1-F^s(x_N^s)}$, respectively, can be used as upper and lower bounds for the buyer agents' optimal reservation value (given the seller agents' reservation value). Similar bounds can be found for the sellers' optimal reservation value.*

Sketch of Proof: For the upper bound, assume reservation value x_N^b satisfies the above condition. Substituting x_N^b in (2) will yield: $V^b(x_N^b) = 0 < x_N^b$ And thus, using theorem 1 we can conclude that x_N^b is an upper bound for the optimal reservation value. The lower bound is valid simply because the overall utility function is concave (Theorem 1) and equation (4) holds. □

At this point we have sufficient knowledge to sketch the graph of the buyer agents' expected utility as a function of the reservation value used, $V^b(x_N^b)$ (which reflects similar characteristics to the seller agents' graph $V^s(x_N^s)$). The basic structure of the curve is given in Figure 3. Thus, an efficient algorithm for calculating an agent's optimal reservation value, up to any precision ρ, given the strategy of the opposite type agents and the number of simultaneous interactions used can be suggested. This algorithm will later be used as part of the mechanism for extracting the equilibrium reservation values.

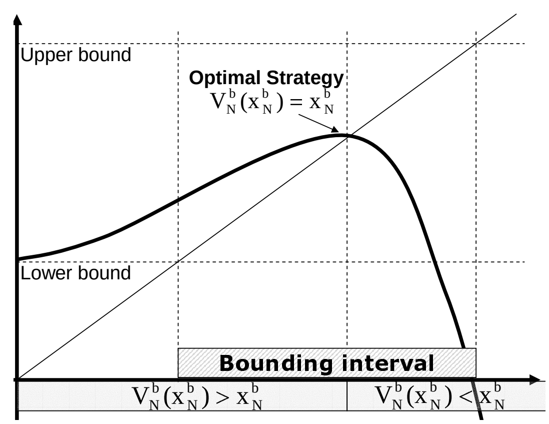

Fig. 3. Specific characteristics of an agent's expected utility function $V^s(x_N^s)$

Algorithm 1 *An algorithm for calculating the optimal reservation value x_N^b for the buyer agents.*

Input: *ρ - precision level; N - number of simultaneous interactions used by the buyers; x^s_N - seller agents' reservation value.*

01. ***if*** $\frac{dV^b(0)}{dx^b_N} < 0$, *return (0);*
02. Set x_{upper} and x_{lower} according to theorem 3;
03. Set $d = (x_{lower} - x_{upper})/2$;
04. Set $x = (x_{lower} + x_{upper})/2$;
05. Calculate $V^b(x)$ using Equation (2);
06. ***if*** $\mid V^b(x) - x \mid > \rho$ *then set d=d/2;*
07. ***else*** *return(x);*
08. ***if*** $V^b(x) > x$ *then set x=x+d;*
09. ***else*** *set x=x-d;*
10. goto 5;

The algorithm will always reach the agent's optimal reservation value, up to any precision ρ, in a finite number of steps. Step 01 gives an immediate result if the optimal strategy is to accept any agent[6]. Based on theorem 3, the optimal reservation value is bounded in the interval (x_{upper}, x_{lower}). Using theorem 1 we can determine if the optimal reservation value is bigger or smaller (steps 08-09) and thus refine our bounding interval (steps 05-06). Notice that for all finite distribution functions the algorithm uses a binary search over a bounded interval. The complexity of the main calculation (step 05) in the loop is determined by the distribution functions. Some functions (e.g. normal distribution) require approximation, while others (exponential, uniform, etc.) only entail a simple direct calculation. A similar algorithm can be suggested for the seller agent, replacing the calculation in step 5 with the appropriate modification for Equation (2).

Before completing this section we would like to emphasize that in a given sample there may be several seller agents suggesting utilities that might be greater than the buyer agent's reservation value. Thus buyer agents can improve their expected utility by considering committing also to the next best seller agents in the sample, upon receiving a rejection from the best seller agent in the sample. In this case, we need to redefine the c.d.f and p.d.f of acceptance and rejection for the buyer agents and the seller agents. Even though we are unable to present our analysis for this specific case, due to lack of space, we do wish to emphasize that the number of optimal simultaneous interactions, N, in this scenario might be different than in the regular model presented earlier. Nevertheless, this variant further improves both buyer and seller agents' performances. It reduces the overall search costs for the buyer agents, and for the seller agents it increases the probability of being accepted by the buyer agents, even if these seller agents are not associated with the highest utility in the buyer agents' sample.

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[6] If the calculation is not immediate, it can be skipped and the algorithm will eventually return the correct reservation value for this case, $x^b_N = 0$.

4.2 Finding the Equilibrium Reservation Values

The equilibrium in our model can be described by a set (N, x^b_N, x^s_N) where the buyer agents cannot gain a better utility by changing N and\or x^b_N and the seller agents cannot gain a better utility by changing x^s_N. Using the analysis given in the former section, we can now combine the reactions of both types of agents to changes in the other agents' reservation value, towards equilibrium.
Notice that an important result from (7), is that an agent's reservation value decreases as a function of the reservation value used by the agents of the opposite type, e.g. for any specific N we obtain that:

$$\frac{dx^b_N}{dx^s_N}, \frac{dx^s_N}{dx^b_N} < 0 \qquad \lim_{x^s_N \to \infty} x^b_N = \lim_{x^b_N \to \infty} x^s_N = 0 \tag{8}$$

This behavior is illustrated in Figure 4. From (8) we conclude that at least one equilibrium exists (in the extreme case, we obtain an equilibrium where agents of a specific type or of both types accept any agent of the opposite type). In some cases we can be sure that there will be a single equilibrium (for example, for the uniform distribution function). However, theoretically, a general distribution function might produce several equilibria with uncertainty regarding the identity of the one that will eventually be used. None of these equilibria dominate the other for both agent types (buyer agents and seller agents). Also notice that if there is more than one equilibrium, then the total number of equilibriums is odd. The research of multiple non-dominating equilibriums in game and agents theory is quite rich [6], and thus we leave this case for future research. Within this context we would prefer to limit the rest of the paper to the scenario where the agent faces a singular equilibrium.

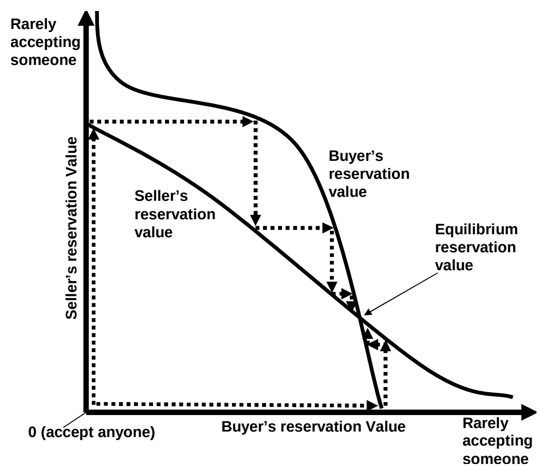

Fig. 4. Agents' reaction curves

An equilibrium reservation value, for any given N, can be calculated by setting one of the agent types' reservation value to 0, and sequentially calculating the optimal reservation value of the two agent types in turns (using either a direct calculation, or algorithm 1), based on the last reservation value calculated in former stage. An illustration of this process is given in Figure 4. Such calculation sequences always converge as each agent increases and decreases its reservation value (as a reaction to the changes in the other agent's reservation value) in a decreasing rate, in each subsequent stage of the process detailed.

Notice that the agent can check the singularity of the equilibrium found, simply by repeating the process while initializing the opposite agent type to zero. If the same equilibrium is reached when starting from both directions, then this is a singular equilibrium.

Figure 5 illustrates the changes in the strategies of agents of the two types, for the specific environment detailed in Figure 1. From this figure we can clearly see that the singular equilibrium point is where buyer type agents use $x_4^b = 65.6$ and seller type agents use $x_4^s = 9.2$. This is in comparison to the (46.5,46.5) equilibrium reservation values, associated with the traditional two sided search model (for $N = 1$), obtained from the middle curve in figure 1 by calculating $V_1(x) = x$. Recall that in Theorem 1 we obtained $V^b(x_N^b) = x_N^b$ (and similarly for the seller type agents), thus in the former scenario, the buyer type agents increase their revenue at the account of seller type agents. This is always true since the buyer agents become more selective, thus decreasing the probability for seller agents to be accepted in a given encounter (which increases the number of search rounds for seller agents, resulting in increased search costs for them).

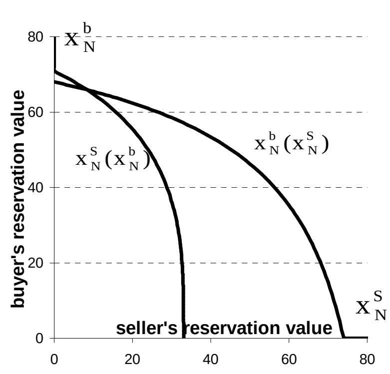

Fig. 5. Equilibrium analysis (N=4)

Now that agents of both types are capable of calculating the equilibrium strategies, given the number of simultaneous interactions, N, we move on to handle the buyer agents problem of setting the optimal value N^*. Figure 6, illustrates the equilibrium expected utility for buyer agents, as a function of the sample size N they set, for the sample environment outlined earlier. In this case, the optimal expected utility will be obtained when using N=11. Thus using the simultaneous search method was beneficial for buyer agents.

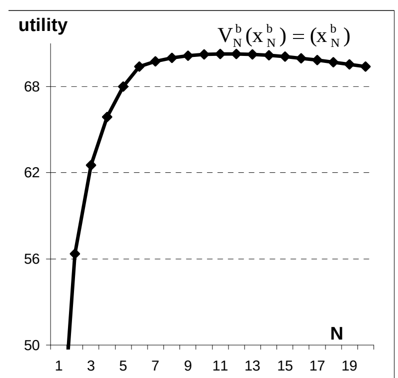

Fig. 6. Extracting best N

As suggested earlier, any buyer agent is interested in the sample size N^*, that will produce the best equilibrium utility for it. This task requires bounding the range of possible values for N^*. The following theorem suggests such an upper bound (we can use $N = 1$ as a lower bound for N^*).

Theorem 4. *The upper bound value for N^* is the solution to the equation:* $N = max(\frac{E[U^s]}{\alpha_s}, \frac{E[U_N^b]-\alpha_b}{\beta})$.

Sketch of Proof: First, we will prove the following Lemmas 1-3.

Lemma 1. *(1)* $\frac{dE[U^b_N]}{dN} > 0$ *(2)* $\frac{dE[U^b_N]}{d^2N} < 0$

Sketch of Proof: (1) Simply split every sample of size N+1 into two samples of size N and 1. Taking the maximum of these two samples, the new random variable $max(U^b_N, U^b)$ will yield a better expected utility. (2) Formulating the explicit expression $E[U^b_{N+2}] - 2E[U^b_{N+1}] + E[U^b_N]$ and integrating by parts we obtain an expression which is always negative, for any $N \geq 1$. □

Lemma 2. *For any N satisfying* $N > \frac{E[U^s]}{\alpha_s}$*, the seller's utility decreases in N.*

Sketch of Proof: Substituting $N = N' > \frac{E[U^s]}{\alpha_s}$ in the appropriate seller's modification for (1), and using integration by parts, we find that the derivative of $V^s(x^s_N)$ is negative for $x^s_N \geq 0$. □

Lemma 3. *For any N which holds:*
$N > max(\frac{E[U^s]}{\alpha_s}, \frac{E[U^b_N]-\alpha_b}{\beta})$*, the buyer agents' utility decreases in N.*

Sketch of Proof: From Lemma 2, we ascertain that in this case the seller agents will accept any buyer agent, thus $(1 - F^s(x^s_N)) = 1$. Substituting $N = N' > \frac{E[U^b_N]-\alpha_b}{\beta}$ in (1), and using integration by parts, we find that the derivative of $V^b(x^b_N)$ is negative for $x^b_N \geq 0$. □
Coming back to the proof of Theorem 4, notice that for any N' satisfying the inequality of the theorem, all agents' equilibrium strategy will be to accept any agent of the other type, and their expected utility decrease in N'. Also notice that since $E[U^b_N]$ is concave (according to Lemma (1)) and $\alpha_b + \beta N$ is linear, there is always a value N which satisfies $N = \frac{E[U^b_N]-\alpha_b}{\beta}$ and for every $N > \frac{E[U^b_N]-\alpha_b}{\beta}$, $N + 1 > \frac{E[U^b_{N+1}]-\alpha_b}{\beta}$ must also hold. □

Once we have bounded the range of possible values for the optimal number of simultaneous interactions to be used by the buyer agents, we can suggest a simple algorithm for finding N^*, given the environment characteristics (distribution functions and search cost parameters).

Algorithm 2 - *An algorithm for finding the optimal number of simultaneous interactions,* N^**.*

01. Set N_{upper} *according to Lemma 3*
02. ***For*** *(N=1;N <* N_{upper}*;N++) calculate the utility associated with the equilibria* $V^b(x^b_N)$ *using Algorithm (1), and the appropriate convergence mechanism that was described earlier in this section.*
03. ***Return*** *the N associated with the maximal utility calculated in 2;*

Notice Algorithm 2 is finite and will always yield N^*, as it scans all integers over a bounded interval. Nevertheless, the complexity highly depends on the characteristics of the utilities distribution functions.

Before continuing to the conclusions section we would like to report that we have also examined a model where all agents are capable of using the combined simultaneous search. Though it is not used in current markets, we do see room for such model in future C2C marketplaces, where seller agents will use more proactive methods to approach buyer agents. In this case the equilibrium characteristics are highly influenced by the structure of resources both agents types are required to invest during search. Other than the additional complexity derived from a process where agents of both types change both N and the reservation value, simultaneously, this case also requires a mechanism for resolving deadlocks that may occur in these many-to-many communication scenarios.

5 Discussion

In this paper we have presented a thorough analysis of the two-sided search model when agents of a specific type make use of simultaneous search interactions in order to improve their equilibrium revenue. The capability for using such search technique is inherent in the infrastructure of autonomous information agents. Furthermore, as demonstrated in section 4, there is a strong incentive for the agent to use such technique in many different environment settings. We emphasize that the agent's utility will never decrease when using our proposed mechanism. As the agent can control the number of simultaneous interactions used in each search round, in the worst case scenario, the proposed calculations will indicate that the optimal number of interactions is 1, thus the expected utility will be identical to the case where the traditional pure sequential method is used. In fact, the latter method is actually a specific case of our general model, using a single interaction over each search round.

Obviously the optimal number of interactions to be used is highly correlated with the ratio of the fixed and variable costs of the agents' search. Increasing the number of interactions suggests a complex tradeoff for each agent. On one hand, it can reduce the average cost of evaluating another agent (as the fixed cost is shared). On the other hand, the agent risks having spare interactions over the last sample taken, since the partner eventually selected could have been reached by sampling fewer agents if a single interaction method had been used. As the fixed cost becomes more dominant in the overall cost structure, the agents will find it more beneficial to use more simultaneous interactions. In the absence of any fixed costs the agents will obviously use the traditional pure sequential search method, and if the variable cost is negligible, the agents will strive to interact with as many other agents as possible in each search round. The nature of MAS applications suggests considerable costs which can be categorized as fixed costs. The most trivial is the agent's maintenance cost per time unit, when operating in the environment. Other fixed costs include self advertisement, fixed batch processes and possibly costs of reporting results to the user after each search round. Thus the new method is highly applicable and beneficial for MAS environments.

We show the special characteristics of the agents' optimal strategies, and the derived equilibrium. We also proffer efficient tools for calculating the optimal number of simultaneous interactions in each search round.

Though we focus on implementing the new search method for agents of a single type, the basic analysis and methodology can be widely used in exploring the dynamics of future models which will also combine the use of proactive simultaneous search by agents of the other type. A first step towards this direction can be found in [15]. We also see great importance in understanding the changes in such models when the agents can negotiate over the division of the overall utility.

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