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Electric Vehicle Charging Strategy Study and the Application on Charging Station Placement

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Abstract Optimal placement of charging stations for electric vehicles (EVs) is critical for providing convenient charging service to EV owners and promoting public acceptance of EVs. There has been a lot of work on EV charging station placement, yet EV drivers' charging strategy, which plays an important role in deciding charging stations' performance, is missing. EV drivers make choice among charging stations according to *various factors*, including the distance, the charging fare and queuing condition in different stations etc. In turn, some factors, like queuing condition, is greatly influenced by EV drivers' choices. As more EVs visit the same station, longer queuing duration should be expected. This work first proposes a behavior model to capture the decision making of EV drivers in choosing charging stations, based on which an optimal charging station placement model is presented to minimize the social cost (defined as the congestion in charging stations suffered by all EV drivers). Through analyzing EV drivers' decision-making in the charging process, we propose a $k-$Level nested Quantal Response Equilibrium charging behavior model inspired by Quantal Response Equilibrium model and level-k thinking model. We then design a set of user studies to simulate charging scenarios and collect data from human players to learn the parameters of different behavior models. Experimental results show that our charging behavior model can better capture the bounded rationality of human players in the charging activity compared with state-of-the-art behavior models. Furthermore, to evaluate the proposed charging behavior model, we formulate the charging station placement problem with it and design an algorithm to solve the problem. It is shown that our approach obtains placement with a significantly better performance to different extent, especially when the budget is limited and relatively low.

Keywords Electric vehicle · Charging strategy · Game theory · Facility placement

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1 Introduction

Electric vehicles (EVs) are attracting growing interest from the public in recent years. Many countries have started investing on EVs to mitigate the shortage of fossil fuels and the serious air pollution caused by traditional combustion vehicles. A critical issue that limits the promotion of EVs is their limited battery capacity, which brings mileage anxiety for drivers. Consequently, the EV charging stations, which can support EVs with fast and convenient charging (between 20 to 30 minutes, around 12 times faster than charging with domestic electricity [1]) is important for the successful boosting of EVs. With integrated network of charging stations, EV drivers can select the most suitable and convenient one to use according to their preference. Such facility support would increase the willingness of the public to accept EVs. On the one hand, characteristics of charging stations (e.g., the location and size) influence the charging behavior of EV drivers; in turn, the choices of EV drivers affect the performance of charging stations (e.g., the length of the queue). Thus, it is important to study the interrelationship between EV drivers' charging behavior and the performance of charging stations, which should be furthermore considered in the planning and construction process of charging stations.

While there has been a number of studies [2–5] on Charging Station Placement Problems (CSPP), only a few of them [2,5] take into consideration the influence of EV drivers' charging behavior. Moreover, among a few studies that mention EV drivers during solving CSPP, their behavior models are based on rather simple assumptions. There is lack of comprehensive study of EV drivers' preference over different factors and/or they assume that EV drivers are fully rational. Although there are some other studies on EV drivers' charging behavior or patterns without considering the charging station placement problem [6–9], they focus on statistics of their charging time, frequency and peak demand etc. None of them have studied the decision making process of EV drivers about choosing charging stations in the charging process.

In this paper, we propose a realistic k-Level nested Quantal Response Equilibrium (k-Level QRE) charging behavior model, which is the first contribution of this work. In the proposed model, with different levels of rationality, EV drivers try to minimize the charging cost and compete with each other over limited resources for charging. Our k-Level QRE charging behavior model is inspired by the QRE model [10] and level-k thinking model [11]. To the best of our knowledge, we are the first to study EV drivers' specific charging behavior.

Our second contribution is that we formulate the charging station placement problem with the k-Level QRE charging behavior model and design an algorithm to solve the complex optimization problem. We utilize the approximate derivative and design a gradient descent based approach.

The third contribution of this work is that we design a set of simulations of charging scenarios and collect data from human players to learn the parameters of different behavior models and compare the fitting results of them. The comparison result proves that our proposed model can better capture the charging behavior of EV drivers.

The last contribution is that we conduct experimental evaluations to prove the effectiveness of employing the k-Level QRE charging behavior model for CSPP. We compare it with benchmarks with other behavior models. It is shown that our approach for placement significantly outperforms the benchmarks by decreasing the EV drivers' queuing duration to different extent, especially when the budget is limited and relatively low.

2 Related Work

Due to the rising attention to environment-friendly energy usage in transportation area, electric vehicle (EV) and its charging techniques have been extensively researched in recent years [12–14]. Meanwhile, to support the introduction of EVs, the charging station placement problem (CSPP) has also been widely studied [15, 16, 3, 17–20, 4, 21–24]. They focus on different aspects, including the charging station coverage, the cooperation with power grid, and the travel cost of EV drivers to access the charging stations etc., to optimize the location and/or size of charging stations. Nevertheless, they lack enough attention on the influence from the participating EV drivers on the performance of the charging stations. Among a few studies that consider EV drivers' charging strategies, He et al. [2] use a multi-nomial logit model to model EV drivers' charging route distribution. However, they fail to explain why the drivers' behavior would form the distribution and how to decide the parameters in the logit model. Xiong et al. [5, 25] assume that the drivers are fully rational in the charging game and would form *Nash equilibrium* in choosing charging stations, which is usually impractical in real-world scenarios.

While perfect rationality has been extensively studied and used to model players' decision-making in congestion games [26, 27], it is not the best solution for the charging game that we want to study. Nash equilibrium (NE) in a game is defined as the state where no player can improve his/her utility by unilaterally changing his/her own decision. While the number of players goes to infinity, NE converges to the Wardrop user equilibrium (UE), i.e., whichever choice used by the players has the same and maximum utility. However, the assumptions in perfect rationality are usually impractical in reality due to (1) players' lack of accurate information (on others' behavior) and (2) limited computational ability. Bounded rationality is first proposed by Simon [28], where players tend to seek a satisfactory solution rather than an optimal one. However, the qualitative definition of "satisfactory solution" does not specify its distance from the optimal solution and thus it is hard to quantitatively evaluate it for specific problems. Moreover, the existence of Bounded Rational User Equilibria (BRUE) makes the solution space a non-convex set.

To model the bounded rationality of human players, Mckelvey and Palfrey [10] propose quantal response equilibrium (QRE). QRE specifies a set of mixed strategies for each player while assuming a random perception error in utility estimation. A typical QRE formation is the logit equilibrium based on a presumed error distribution, i.e., i.i.d. Gumbel distribution.

$$p_i = \frac{e^{\lambda u_i}}{\sum_j e^{\lambda u_j}}$$

Note that the subscript i denotes a specific choice, u_i is the utility of choice i, and p_i is the probability of using choice i. However, the hyper-parameter — rationality level λ defined in QRE can be any value from 0 to ∞, and it may vary from case to case. This character hinders the application of QRE to real-world problems. Some work (e.g. [29]) discovers that value of λ is largely dependent on specific problem structure, but there is no further research on how it is influenced. Thus, the hyper-parameter λ is usually carefully studied for specific problems and applications.

Human behavior has also been valued and extensively studied in the economic research community. Prospect theory, a Nobel-prize-winning theory is a classic behavior economic theory proposed by Daniel Kahneman and Amos Tversky [30]. As shown in

Figure 1, people's prospect utility for a certain decision is influenced by a reference point, as well as the loss or gain versus it. Another behavior model, cognitive hierarchy model (which is well known as level-k thinking model [11]) assumes that players act with different levels of rationality.

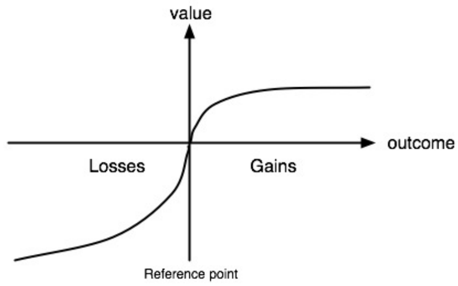

Fig. 1: The value function of prospect theory

Some existing work has focused on EV drivers' charging behaviors or patterns. Smart and Schey [6] collect and analyze data collected from an EV project. Their data show some statistics of EV users, including the driving distance, charging frequency and the place to charge (home or charging stations) etc. Franke and Krems [7] focus on studying the charge level at which people recharge. Azadfar et al. [8] also present some statistics of EV drivers' recharging behavior such as driving distance between charging activities etc. Quirós-Tortós et al. [9] use probability distribution functions to analyze EV drivers' start charging time and peak demand etc. to understand the interaction between charging activities and the power grid. However, to the best of our knowledge, there is no existing work studying EV drivers' charging profile and strategy.

In this work, we formulate the charging process of EV drivers as a charging game; study and describe the bounded rationality of EV drivers in charging activities with a k-Level QRE model (which is inspired by the QRE model and level-k thinking model); and integrate the obtained realistic charging behavior model into the CSPP formulation to improve the charging station performance.

3 Charging Game and Equilibria

In this section, we first describe the EV charging scenario and model it as a charging game. Then we discuss three kinds of different equilibrium concepts for the charging game, including two state-of-the-art equilibria and the k-Level QRE model proposed in this work.

3.1 Charging Scenario and Charging Game Formulation

When EV drivers need to charge their EVs, they are usually faced with multiple choices, where each is a charging station with some specific characteristics. These characteristics may include the distance to the station, the charging fare to pay for charging and the

Notation	Meaning
$\mathcal{N} = \{1, ..., n\}$	the set of zones; if there is a charging station in zone i, it is named the ith charging station
τ	the portion of EV owners that use charging stations
x_i	the number of chargers in the ith charging station; $x_i = 0$ means there is no charging station in zone i
E_i	the number of EV owners in zone i; they are treated as identical group i players of the charging game
$\mathbf{p}_i = \{p_{ij}\}$	the mixed strategy of EV owners in zone i, where p_{ij} is the probability they charge in the jth charging station
$\mathbf{P} = \{\mathbf{p}_i\}$	the strategy profile of all EV owners
$\mathbf{P}^\star$	An equilibrium strategy profile
$\mathbf{P}^\star_{-i}$	An equilibrium strategy profile of all players except group i players
C_i	the charging cost of all EV owners in zone i
c_{ij}	the unit charging cost for EV owners in zone i to charge in the jth charging station
t_{ij}, d_{ij}	the travel time and distance between zone i and the jth charging station
f_j	the charging fare of the jth charging station
q_j	the queuing duration in the jth charging station, which is a function of the strategy profile $\mathbf{P}$
μ	the time used to charge one EV
y_j	the number of EV owners that use the jth charging station under strategy profile $\mathbf{P}$, $y_j = \sum_i E_i \tau p_{ij}$
w_t, w_d, w_f, w_q	the weights of travel time, distance, charging fare and queuing duration in function c_{ij}
λ	the rationality parameter of QRE and k-Level QRE models
γ_l	the proportion of level-l players in the k-Level QRE model, $\sum_{l=1}^{k} \gamma_l = 1$
$\mathbf{p}_i^l = \{p_{ij}^l\}$	the strategy of level-l players in zone i
SC	the social cost
B	the budget
$\mathcal{S} = \{\mathbf{x}^0\}$	the set of initial searching points, each of which is a valid placement
N_I	the maximum number of iterations of Algorithm 1
N_S	the maximum searching step size (integer) of Algorithm 1

Table 1: Notations

queuing condition in the station etc. Specifically, considering the queuing condition, it is straightforward to see that more people selecting the same station would lead to longer queuing duration. In a sense, people are competing with each other for the resource — the charging stations and they are self-interested in this process. Therefore, we define a charging game to model the interactions among them.

Let us first take a look at the charging scenario. We consider the EV charging problem in such an environment. Assume that there is an EV driver population scattered in the city. According to the *geographic condition*, residential distribution and city plan, the city can be divided into a set $\mathcal{N} = \{1, ..., n\}$ of zones[1]. Each zone $i \in \mathcal{N}$ has E_i residents that own EVs and a part $\tau \in (0, 1)$ of them use charging stations instead of home electricity to recharge their EVs. Suppose there is at most one charging station in each zone i and we name it the ith charging station, whose size is x_i (i.e., there are x_i chargers and at most x_i EVs can be served in the meantime) and location is decided by the government according to various factors (e.g., the land property and city planning requirement etc.)[2]. Then the EV charging problem is for each EV driver to make decision about where to charge the EV. The charging game has following components.

- ***Player.*** The EV drivers are players of the charging game.
- ***Strategy, mixed strategy and strategy profile.*** A strategy for a player is to use one accessible charging station. For example, a player in zone i can charge in the jth charging station. Considering that players in the same zone have the same accessibility to charging stations, we treat them as identical players. The EV drivers in zone i are thus called *group i players*. We then focus on the mixed strategy $\mathbf{p}_i = \{p_{ij}\}$, i.e., the strategy distribution of players in each group i, where p_{ij} is the probability that players in zone i charge in the jth charging station. The strategy profile of the charging game is denoted by $\mathbf{P} = \{\mathbf{p}_i\}$, i.e., the mixed strategy of all groups.
- ***Cost.*** Considering that charging in different charging stations bring the same utility for EV drivers, i.e., having the EV recharged, we assume that EV drivers of each group i only consider their cost C_i (details to be discussed later in this section) when making decision in the charging process[3].
- ***Equilibrium.*** Players are self-interested in the charging game, thus they are always searching for better strategy to decrease the charging cost with their best ability. An equilibrium strategy profile $\mathbf{P}^\star$ is the stable state of the game where no player has the incentive to unilaterally deviate from their current strategy.

We then discuss the cost definition in the charging game. Considering the charging process, an EV driver needs to drive from home to the charging station, (probably) queue in the charging station for some time and pay the charging fare. Thus, we decide to include the following factors in the charging cost function[4].

- **Travel time** t_{ij}: the time of driving between zone i and the jth charging station. We assume that the travel time is similar for going to and coming back from the charging station.
- **Travel distance** d_{ij}: the distance between zone i and the jth charging station.

[1] This is inspired by the city plan of Singapore (`http://www.propertyhub.com.sg/singapore-district-guide.html`). Based on the zoning assumption, residents in the same zone are living relatively close. Although identifying the specific location and treating each of them as a different player would be closer to the real-world scenario, relatively unrealistic for formulating and solving the optimization problem. Thus, we make the comprise and treat them as a group of identical agents.

[2] Readers might wonder why only *one* charging station is considered in one zone. The reason is that in case there are multiple charging stations in one zone, we can always divide the zone into a number of new zones, each with one charging station.

[3] Under some circumstances, EV drivers might have different *benefits* while charging their EVs in different charging stations (e.g., getting access to other facilities). In that case, our model can be extended by deducting the *benefit* in the cost function.

[4] We list the most common factors that influence EV drivers' charging cost. While other factors may make a difference in some special scenarios, the model can be extended accordingly.

- **Charging fare** f_j: the money to pay in the jth charging station. We assume a fixed fare for all players that use the same charging station.
- **Queuing duration** $q_j(\mathbf{P})$: the time to wait in line in the jth charging station, which depends on the size x_i of the charging station and the number of EV drivers $y_i = \sum_i E_i \tau p_{ij}$ that use it. Note that the queuing duration q_j in the charging station of zone j is decided by how many EV drivers use it. Thus it is a function of the *strategy profile* $\mathbf{P}$. Similar to [25], we assume that all chargers to be deployed have the same service ability, and each of them averagely takes μ minutes to recharge one EV. The queuing duration q_j for EV drivers in the jth charging station is

$$q_j(\mathbf{P}) = \frac{\mu y_j}{2x_j} = \frac{\mu \sum_i E_i \tau p_{ij}}{2x_j} \tag{1}$$

A linear of aforementioned factors with corresponding weights is used to formulate charging cost C_i.

$$C_i(\mathbf{P}) = E_i \tau \sum_j p_{ij} c_{ij}(\mathbf{P}) \tag{2}$$

$$c_{ij}(\mathbf{P}) = w_t t_{ij} + w_d d_{ij} + w_f f_j + w_q q_j(\mathbf{P}) \tag{3}$$

where $q_j(\mathbf{P})$, $c_{ij}(\mathbf{P})$ and $C_i(\mathbf{P})$ mean that the queuing duration q_j and cost C_i are functions of the strategy profile $\mathbf{P}$, and c_{ij} is the unit cost of charging in the jth charging station.

3.2 Nash Equilibrium

Nash equilibrium is widely used in game theory. By assuming that players have the knowledge of all other players' strategies and are fully rational, the Nash equilibrium describes the state where no player can decrease her charging cost via unilateral strategy change. In the charging game, it means: if the jth charging station is used with non-zero probability p_{ij} by group i players, there must be charging cost $c_{ij} = \min_{j'} c_{ij'}$. Otherwise, p_{ij} will decrease while $p_{ij^\star}$ with $j^\star = \arg\min_{j'} c_{ij'}$ would increases. Formally, the Nash equilibrium $\mathbf{P}^\star$ can be denoted as Equation (4), which can further be represented by Equation (5).

$$\mathbf{p}_i^\star \in \arg\min_{\mathbf{p}_i} E_i \tau \sum_j p_{ij} c_{ij}(\mathbf{p}_i, \mathbf{P}_{-i}^\star), \forall i \in \mathcal{N} \tag{4}$$

$$c_{ij}(\mathbf{P}^\star) = \min_{j'} c_{ij'}(\mathbf{P}^\star), \text{if } p_{ij}^\star > 0, \forall i \in \mathcal{N} \tag{5}$$

Note that $\mathbf{P}_{-i}^\star$ denotes the equilibrium strategy profile of all players except group i players.

3.3 QRE Model

With the charging cost function in Equation (3), we can denote the selection distribution of players according to Quantal Response Equilibrium (QRE) model with Equation (6).

$$p_{ij} = \frac{e^{-\lambda c_{ij}(\mathbf{P})}}{\sum_{j'} e^{-\lambda c_{ij'}(\mathbf{P})}} \tag{6}$$

Note that λ is the *rationality parameter* of the QRE model. When $\lambda \to 0$, players tend to be irrational and choose one charging station randomly; when $\lambda \to \infty$, players tend to be rational and choose the option with the lowest cost. In this case, players are actually using the Nash equilibrium.

3.4 k-Level QRE Model

A strong assumption in aforementioned QRE model is that players can form QRE distribution $\mathbf{P}$ according to the actual charging cost $c_{ij}(\mathbf{P})$. In practice, QRE is usually employed when the cost for each strategy is a constant, which is different from our charging game, where the charging cost of a strategy is in turn a function of the strategy distribution (as shown in Equation (3)). In other words, players can hardly know $c_{ij}(\mathbf{P})$ when they are making decision.

With respect to the above characteristic, we propose a k-Level nested Quantal Response Equilibrium (in short, $k-$Level QRE) model to capture the decision making of the EV drivers.

We assume that there are players with k different levels of rationality, which means players from different levels perceive the unknown charging cost differently and make distinct decisions. Specifically, the level-1 players are the least rational players and would form a QRE distribution according to *their direct observation*, i.e., the current queuing duration that they observe. We denote the observable queuing duration for all charging stations as $\mathbf{q}^0 = \{q_1^0, ..., q_n^0\}$, where $q_j^0, j \in \mathcal{N}$ is for the jth charging station. Then, level-1 players would form the following QRE distribution $\mathbf{P}^1$ with each

$$p_{ij}^1 = \frac{e^{-\lambda c_{ij}(\mathbf{q}^0)}}{\sum_{j'} e^{-\lambda c_{ij'}(\mathbf{q}^0)}} \tag{7}$$

Consequently, level-2 players, as they are more intelligent and rational, would think other players are in level-1, anticipate their choice distribution $\mathbf{P}^1$ and perceive the queuing duration as $\mathbf{q}^1(\mathbf{P}^1)$. Note that the queuing duration in charging stations depends on the strategy profile $\mathbf{P}^1$, thus it is a function of $\mathbf{P}^1$. The QRE distribution $\mathbf{P}^2$ formed by level-2 players is then presented with Equation (8).

$$p_{ij}^2 = \frac{e^{-\lambda c_{ij}(\mathbf{q}^1(\mathbf{P}^1))}}{\sum_{j'} e^{-\lambda c_{ij'}(\mathbf{q}^1(\mathbf{P}^1))}} \tag{8}$$

Similarly, level-k players form QRE distribution $\mathbf{P}^k$ as Equation (9).

$$p_{ij}^k = \frac{e^{-\lambda c_{ij}(\mathbf{q}^{k-1}(\mathbf{P}^{k-1}))}}{\sum_{j'} e^{-\lambda c_{ij'}(\mathbf{q}^{k-1}(\mathbf{P}^{k-1}))}} \tag{9}$$

In Figure 2, we can see the illustration of k-Level QRE model's structure while $k = 2$.

We denote the portion of level-l players as γ_l. Then, the actual distribution $\mathbf{P}$ of all players' charging choices is decided by the distribution of players in each level and the vector $\boldsymbol{\gamma}$:

$$p_{ij} = \sum_{l=1}^{k} \gamma_l p_{ij}^l \tag{10}$$

where $\sum_{l=1}^{k} \gamma_l = 1$.

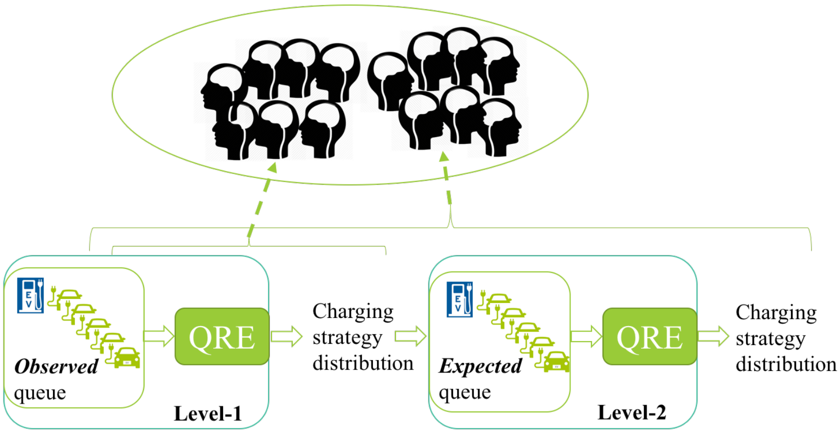

Fig. 2: The k-Level QRE model with $k = 2$

4 Charging Station Placement Problem Formulation and Algorithm

Based on the charging game definitions in the above section, we can then define the Charging Station Placement Problem (CSPP). Actually, the CSPP is a problem to find the best charging station placement, i.e., vector $\mathbf{x} = \{x_i\}$, to optimize a pre-defined objective w.r.t. the charging game equilibrium state.

We use Singapore as a concrete example to demonstrate the process of formulating and solving the CSPP. As we can see from Figure 3, Singapore city is a well-developed metropolitan with the whole territory divided into a number of zones according to the residential condition, geography and nature etc. We divide the island into a set $\mathcal{N}$ of n zones for the CSPP, which is shown with broad lines on the map.

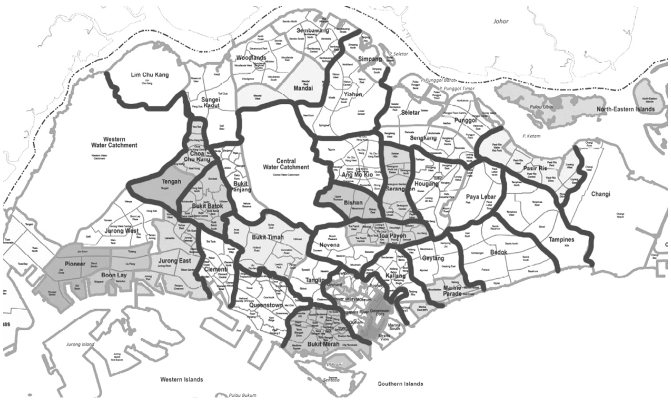

Fig. 3: Zonal division of a target area

We treat the government as the investor to construct the charging stations and they aim to optimize the objective for peak hours, which is the most critic time period for city transportation. To decide the objective of the CSPP, we mainly take two factors into consideration. (1) Congestion issue is a main concern for most big cities in the world [31]. (2) The limited space and relatively long charging time for EVs would make congestion a potential problem for charging stations. Thus, we define social cost SC as the objective to optimize in the CSPP, which is the total queuing duration of all EV drivers[6], i.e., Equation (11).

$$SC = \sum_{i \in \mathcal{N}} \tau E_i \sum_j p_{ij} q_j(\mathbf{P}) \tag{11}$$

Meanwhile, a budget B is introduced to constrain the upper bound of investment from the government, which is the total number of chargers that can be placed in all charging stations. Then, the CSPP is to minimize SC by strategically deciding the optimal charging station placement $\mathbf{x} = \{x_i\}$ with respect to the EV drivers' charging behavior in the charging game.

Specifically, with our proposed k-Level QRE model, the CSPP can be formulated as follows.

$$\min_{\mathbf{x}} \quad SC \tag{12}$$

$$\textbf{s.t.} \quad (1) - (3), (7) - (10)$$

$$\sum_{i \in \mathcal{N}} x_i \leq B, x_i \in \mathbb{N} \tag{13}$$

The CSPP is an integer non-convex optimization problem, finding the global optimum is NP-hard. The number of possible charging station placements is as large as n^B, so it is impossible to enumerate all solutions to find the optimal one. Therefore, we propose an algorithm named MAGD (Algorithm 1) with techniques including multi-start point searching, derivative approximation and gradient descent method to compute an approximate solution.

MAGD solves the charging station placement problem with multiple start points, with each of which, it iteratively finds the best local optimum. Firstly, we randomly generate a set of start points $\mathcal{S}$ (Line 1), where each $\mathbf{x}^0 \in \mathcal{S}$ represents a charging station placement. The optimal object value is initialized as infinity (Line 2). For each start point $\mathbf{x}^0$, the algorithm use gradient decent method to search the corresponding local minimal objective value O^0_{bj} (Lines 3 to 19). We initialize O^0_{bj} by setting $\mathbf{x}$ as $\mathbf{x}^0$ and solve the relaxed CSPP (Line 4). In each of the N_I iterations, for each integer step size from the maximum N_S (> 1) to 1, we first set $\mathbf{x}$ as the current $\mathbf{x}^0$ (Line 7) and solve the optimization problem to get the number of EV users in each charging station, i.e., $\{y_i\}$ (Line 8), then compute the approximated gradient for each x_i (Line 9). The vector $\mathbf{x}$ is then updated by increasing (resp. decreasing) *step* for the x_i with the minimal (resp. maximal) gradient (Lines 10 - 12). The objective for the updated $\mathbf{x}$ is computed as O_{bj} (Line 13), which is used to compare with the current local optimal objective O^0_{bj}. If $O_{bj} < O^0_{bj}$, both $\mathbf{x}^0$ and O^0_{bj} will be updated and we go to next iteration; otherwise, we would change *step* size. After N_I iterations, we compare the local minimal objective O^0_{bj} of the start point with the $O^\star_{bj}$ and update the current optimal solution and objective if $O^0_{bj} < O^\star_{bj}$. By increasing the number of start points and

[6] The reason is that we think the most important factor is the congestion in charging station for this placement problem. But our framework is able to be extended to include other factors.

expanding the searching space, the probability of reaching the global optimal solution would be increased.

The complexity of the MAGD algorithm is $O(2|\mathcal{S}|N_I N_S C_{relax})$ where C_{relax} is the complexity of the relaxed CSPP with fixed variable $\mathbf{x}$ to compute the strategy profile $\mathbf{P}$. With the $k-$Level QRE model, $C_{relax} = O(k|\mathbf{P}|)$, where $|\mathbf{P}| = \sum_i \sum_j 1$ is the size of the strategy space. To find the strategy profile of the relaxed CSPP, we start with computing the level-1 players' strategy with the observable queuing duration (Equation (7)), with which the perceived queuing duration for the level-2 players would be computed and used to compute the corresponding level-2 strategy profile (Equation (8)), and so forth. After getting the strategy profile of players from all k levels, we then use Equation (10) to compute the global strategy profile $\mathbf{P}$. Therefore, the overall complexity of the MAGD algorithm is at most $O(n^2)$ when players in each zone i would charge in any of the n charging stations.

Algorithm 1: MAGD - Multi-start Approximate Gradient Descent Algorithm

Generate $\mathcal{S}$;
$O^\star_{bj} \leftarrow \infty$;
for *each* $\mathbf{x}^0 \in \mathcal{S}$ **do**
    Compute O^0_{bj} with $\mathbf{x} \leftarrow \mathbf{x}^0$;
    for $I_{ter} = 1 : N_I$ **do**
        for $step = N_S : -1 : 1$ **do**
            $\mathbf{x} \leftarrow \mathbf{x}^0$;
            Solve CSPP with fixed $\mathbf{x}$ to get $\{y_i\}$;
            $\bigtriangledown\mathbf{x} \leftarrow \{\frac{-step \cdot y_i^2}{x_i(x_i+step)}\}$;
            for $i \in \mathcal{N}$ **do**
                $x_i \leftarrow x_i + step$, if $\bigtriangledown x_i = \min_{i' \in \mathcal{N}} \bigtriangledown x_{i'}$;
                $x_i \leftarrow x_i - step$, if $\bigtriangledown x_i = \max_{i' \in \mathcal{N}} \bigtriangledown x_{i'}$;
            Compute O_{bj} with updated $\mathbf{x}$;
            if $O_{bj} < O^0_{bj}$ **then**
                $\mathbf{x}^0 \leftarrow \mathbf{x}$;
                $O^0_{bj} \leftarrow O_{bj}$;
                Goto next I_{ter};
            else
                Goto next *step*;;
    if $O^0_{bj} < O^\star_{bj}$ **then**
        $\mathbf{x}^\star \leftarrow \mathbf{x}^0$;
        $O^\star_{bj} \leftarrow O^0_{bj}$;
return $O^\star_{bj}$ and $\mathbf{x}^\star$;

5 User Study Design

To learn the charging cost function and the level of rationality of EV drivers, we design a set of user studies to simulate the charging scenarios and collect data from human players.

We present charging scenarios for players with abstracted information as shown in Figure 4[7]. The interface includes (1) a grey scale map as background, (2) the start point[8] marked with red circled S where EV drivers (players) stay, (3) the number of EV drivers at the start point that will go for charging at the same time, (4) the candidate charging stations marked with purple icons and named as $CS_i, i \in \{1, 2, ...\}$, (5) the charging fare in \$ at each charging station circled near the corresponding charging station, (6) the hint for queuing duration, (7) the charging routes from the start point to each candidate charging station, along which the travel time and distance are denoted in *min* and *km* respectively, and (8) a table with text information below the map.

The travel time and the distance from the start point to each of the charging stations are assumed to be constants. To visualize the travel speed in the charging route, inspired by Google Maps, we use four colors to draw the travel routes (respectively representing travel speed from fast to slow. All candidate charging stations are assumed to be located beside a shopping center[9]. The EV drivers at the same start point will charge at the same time, thus they would cause congestion in the charging stations. The EV drivers in a charging station with x EV drivers choosing it would wait for x mins on average before starting charging.

With all the information provided, a player is able to see the difference between different charging choices and then make his/her charging decision. For example, as shown in Figure 4, if a player at the start point S chooses the charging station CS_1, meanwhile there are 12 other EV drivers that also select CS_1, the player would travel 13 minutes, 9 kilometers, pay 1\$ and queue 13 minutes before charging.

5.0.1 Environment Setting

To study the charging behavior of EV drivers, we design two different charging scenario settings $\{I_1, I_2\}$ respectively with: (1) one start point and two candidate charging stations (I_1); and (2) one start point and three candidate charging stations (I_2). For each setting, we carefully design 6 scenarios with different parameters. The basic idea is to provide distinct scenarios to avoid over-fitting. The total 12 charging scenarios are separated into two groups $\{G_A, G_B\}$, each with 3 from I_1 and another 3 from I_2.

To decide the parameters in the charging scenarios, we first randomly generate $1,000$ settings respectively for I_1 and I_2, each of which with travel time, distance,

[7] The design of the game has been performed in several iterations with studies on human players to ensure that they are aware of all the games' parameters.

[8] We design the charging games to capture the play behavior when competition exists in the charging scenario by setting one start point but multiple EV drivers. Although there are EV drivers from other start points in real-world scenarios, they form the equilibrium after repeating charging activities in a long term. Setting a too complicated charging game would make it unrealistic for players to fit in a short time.

[9] In real-world application, a charging station can be located in other positions. In the experiments, we say that a charging station is next to a shopping center to provide a simulated scenario for players.

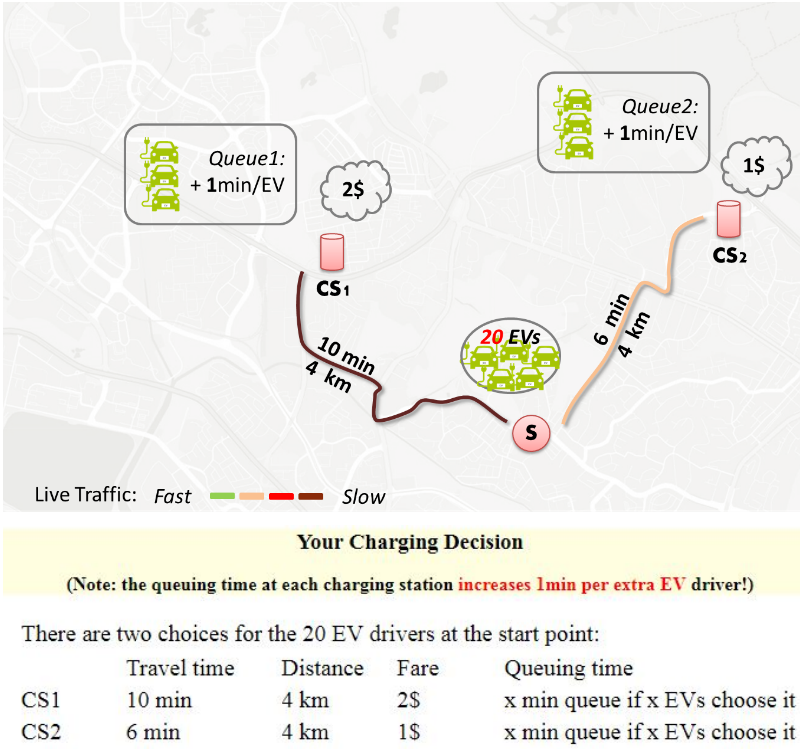

Fig. 4: An example of user study interface

charging fare and the number of EVs all randomized from a given interval/set. Then we use k-means clustering to class the settings into 6 clusters and select one from each cluster in two steps as follows. Table 2 presents all settings got the 12 charging games.

1. Find the maximum d_{max} and minimum d_{min} distance between the settings' parameter vectors in a group and the corresponding group center. Set the radius for searching as $0.5 * d_{max} + 0.5 * d_{min}$.
2. Randomly select a setting from the cluster in the circle with the cluster center and distance radius.

5.0.2 Charging Game

Each invited human player is randomly directed to G_A or G_B. They are asked to make a choice in each of the 6 different scenarios *for once*.

We put the user studies on a virtual machine build on Microsoft Azure platform. Players are invited to access the user studies via a link. They would first see the introduction with a video and text explanation, which clearly explain how the charging scenarios are set and what decisions they are supposed to make. After acquiring the preliminary information, players would see a toy example on the next page, which is used to ensure that the participants have fully understood the user study. Until then players would formally start the charging games.

Table 2: Parameters of the 12 charging games (travel time t, distance d and charging fare f respectively in $min, km, \$$)

Game	$CS_1(t,d,f)$	CS_2	Game	$CS_1(t,d,f)$	CS_2	CS_3
1	8, 6, 4	8, 6, 1	7	8, 7, 3	8, 8, 2	9, 8, 1
2	18, 6, 2	8, 6, 2	8	7, 6, 3	8, 7, 2	7, 7, 3
3	12, 5, 2	12, 12, 2	9	4, 3, 4	6, 3, 3	10, 8, 2
4	8, 5, 2	12, 12, 2	10	4, 3, 2	6, 3, 3	10, 8, 3
5	9, 5, 1	8, 6, 3	11	10, 8, 2	10, 9, 1	1, 1, 3
6	18, 7, 4	9, 10, 1	12	9, 8, 4	12, 9, 1	2, 1, 2

5.1 Model Fitting

We employ the Maximum Likelihood Estimation (MLE) to learn the parameters $\mathbf{w}$. For a charging scenario S with M records from the players, the logarithmic likelihood of $\mathbf{w}$ is

$$\log L(\mathbf{w}|S) = \sum_{i=1}^{M} \log p_{cs(i)}(\mathbf{w}) \tag{14}$$

where $cs(i)$ is the charging station selected in the sample i. Assuming that there are B charging stations (strategies) for the players and the number of players that choose to use the j^{th} is M_j, then we have $\sum_j M_j = M$ and

$$\log L(\mathbf{w}|S) = \sum_{j=1}^{B} M_j \log p_j(\mathbf{w}) \tag{15}$$

By substitute p_j with k-Level QRE model, we have

$$\log L(\mathbf{w}|S) = \sum_{j=1}^{B} M_j \log(\sum_{l=1}^{k} \gamma_l p_j^l) \tag{16}$$

As we can see from above function, $\gamma = \{\gamma_l\}$ is another set of unknown parameter. Our approach is to enumerate the value of each γ_l within $[0,1]$ with interval 0.01, and find the best set.

In fact, we design a number of charging scenarios with different environment settings to study players' charging behavior. Thus, we are maximizing the sum of the logarithmic likelihood when learning the parameters. The function *fmincon* of Matlab is used for the maximization.

$$\max_{\mathbf{w},\gamma} \log L(\mathbf{w}) = \sum_{S} \log L(\mathbf{w}|S) \tag{17}$$

6 Experimental Result

We invite $N = 50$ players to participate the charging game, with 25 of them making choices for charging scenarios in group G_A and the rest for group G_B. In this section, we fit the charging behavior models with results from charging games and discuss the results.

For a charging scenario with M EVs set for the start point, the actual queuing duration in a candidate charging station i with N_i players selecting it is calculated as $q_i = M \cdot N_i/N$ min.

We use *Python* language and *PyOpt* package to solve the electric vehicle charging station placement optimization problem. We compare the result with (1) using QRE and (2) an existing method from [32], which assumes EV owners form Nash equilibrium in the charging game, i.e., the charging cost of each choice should be the same and minimum if the choice is selected by any player.

6.1 Fitting Results of Charging Behavior Models

We fit Nash equilibrium ("NE") model (Equations (4) and (5)), QRE model (Equation (6)) and k-Level QRE ("k-LQRE") models (Equation (10)) of different k values ($k \in \{1, 2, 3, 4\}$) with the experimental data collected from human players. Table 3 presents the fitting errors' mean and standard deviation for different behavior models. *MAE* is the mean absolute error ($\in [0, 1]$) and smaller *MAE* value means better fitting performance. R^2 is the mean squared error ($\in (-\infty, 1]$ for non-linear models). A R^2 value closer to 1 means a better fitted model. D_{KL} — the Kullback-Leibler Divergence is used to evaluate how different are two distributions. Smaller D_{KL} means two distributions are more similar, i.e., the fitted model is closer to the collected data. As we can see, the R^2 value for some of the models is negative, which is basically saying that the mean of the data provides a better fit to the outcomes than do the fitted non-linear function values [33]. Combining the fitting metrics, we can see that the 3−Level QRE model performs best. Since the 4−Level QRE model is quite close to the 3−Level one, we further compare the result of Chi-square test on these two models. The null hypothesis is that the charging strategy profile follows the corresponding behavior model. It is shown that the 3−Level QRE model has a larger p-value 0.96. Thus we select $k = 3$ as the best hyper-parameter for the k-Level QRE model in the charging game. The experiments for charging station placement would be performed with the 3−Level QRE model.

Table 4 shows the learnt parameters for different behavior models, where λ is the rationality parameter when applicable and others are the weights for corresponding factors. Table 5 demonstrates the proportion of players in each rationality level for the k−Level QRE models. For example, when $k = 3$, the results show that 56% of players are in level 1 and their charging decisions are made without considering others' behavior; 22% players think others are in level 1 and take into consideration their strategies; and the rest are in level 3.

6.2 Charging Station Placement

As discussed in Section 4, we can divide Singapore into 23 zones. We present the experimental results on Singapore in this section. We also test the scalability of our proposed algorithm with synthetic data because we might need to apply the proposed approach in bigger cities. In experiments with synthetic data, we increase the number of zones from 20 to 200 with a step size of 20. It turns out that the run time of our approach

Table 3: Learning results from charging games

Models\ Errors - mean(std)	*MAE*	R^2	D_{KL}
NE	0.147(0.063)	-5.452(17.444)	0.124(0.077)
QRE	0.062(0.030)	-0.189(3.366)	0.028(0.027)
1-LQRE	0.064(0.035)	-0.541(4.324)	0.033(0.034)
2-LQRE	0.052(0.033)	0.403(1.407)	0.023(0.025)
3-LQRE	**0.048(0.036)**	**0.833(0.251)**	**0.017(0.014)**
4-LQRE	0.052(0.035)	0.779(0.346)	0.017(0.020)

Table 4: Learning results from charging games

Models\ Params.	λ	w_t	w_d	w_f	w_q
NE	–	0.250	0.250	0.250	0.250
QRE	3.918	0.060	0.028	0.251	0.660
1-LQRE	0.997	0.135	0.058	0.556	0.251
2-LQRE	1.793	0.167	0.046	0.626	0.160
3-LQRE	1.808	0.199	0.006	0.619	0.176
4-LQRE	2.012	0.245	0.000	0.568	0.187

Table 5: Proportion of players in each rationality level

Models\ γ	1	2	3	4
1-LQRE	1	–	–	–
2-LQRE	0.8	0.2	–	–
3-LQRE	0.56	0.22	0.22	–
4-LQRE	0.19	0.00	0.53	0.28

increases with the problem size and the maximum run time of the problem with 200 zones is about 17 hours, which means the proposed approach can handle charging station placement problems in cities as large as 10 times of Singapore. Therefore, we claim that the proposed approach is applicable on real-world scenarios.

Especially, we test our approach with data of Singapore[10]. The main focus is to compare the social cost of different placement plans while assuming that people actually follow the *k*-Level QRE charging behavior model. As presented in Table 6, we compare with 4 benchmarks: (1) the existing work that assumes EV drivers follow Nash equilibrium; (2) quantal response equilibrium; (3) randomized placement and (4) demand-based plan that assign chargers to different zones proportionally according to its number of EV drivers. As we can see from the table, our approach can decrease the social cost to different extent comparing to all benchmarks, especially when the resource is limited and the budget is relatively low.

[10] Parameters are available on `https://drive.google.com/open?id=1K6AYYA_vq6NjYm1jJtSSjcsENwV1ZqdX`

Table 6: Social cost comparison

SC \ Budget	300	400	500	600
3-Level QRE	6947.82	5256.62	4190.56	3523.95
NE	7625.75	5750.31	5001.71	3719.27
QRE	7604.78	5805.69	4462.38	3564.42
Randomized	8877.99	6451.17	5132.83	4249.91
Demand-based	7432.53	5545.14	4443.04	3705.98
Decreased cost VS. NE (%)	8.89	8.59	16.22	5.25
Decreased cost VS. QRE (%)	8.64	9.46	6.10	**1.14**
Decreased cost VS. Randomized (%)	21.74	18.52	18.36	17.08
Decreased cost VS. Demand-based (%)	**6.52**	**5.20**	**5.68**	4.91

7 Conclusion

In this work, we study and model the bounded rational charging behavior of electric vehicle (EV) drivers and apply the behavior model in solving EV Charging Station Placement problem (CSPP). There are several contributions of this work. (1) We propose a k-Level QRE charging behavior model based on the QRE model and the level-k thinking model. The proposed model well captures the bounded rationality of EV drivers in charging activities. (2) We design a series of user studies to simulate the real-world charging scenarios and collect data from human players. Experimental results of fitting different behavior models based on the collected data show that our behavior model outperforms state-of-the-art models. (3) The charging station placement problem is formulated by considering the EV drivers' bounded rational charging behavior. An algorithm is designed to solve this complex integer non-convex optimization problem. (4) To show the efficiency of the proposed charging behavior model, we execute experiments on charging station placement to compare the expected social cost of using different behavior models. The results show that our model significantly decreases the social cost. The EV charging behavior model can also be applied to other related problems. For example, when charging stations have been constructed, governors can use pricing as a method to incentivize the EV drivers' charging decision.

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