# Strategic-negotiation for Sharing a Resource between Two Agents*

Sarit Kraus[1,2] Orna Schechter[1]
[1]Department of Mathematics and Computer Science,
Bar-Ilan University, Ramat-Gan 52900 Israel
e-mail: {sarit,ornash}@macs.biu.ac.il
[2]Institute for Advanced Computer Studies
University of Maryland
College Park, MD 20742

**Abstract**

In this paper, we propose a strategic-negotiation model which enables self-motivated rational agents to share resources. The strategic-negotiation model takes the passage of time during the negotiation process itself into account. The model considers bilateral negotiations in situations characterized by complete information, in which one agent loses over time while the other gains over time. Using this negotiation mechanism, autonomous agents apply simple and stable negotiation strategies that result in efficient agreements without delays, even when there are dynamic changes in the environment. Simulation results show that our mechanism performs as well as a centralized scheduler and also has the property of balancing the resources' usage.

*This material is based upon work supported in part by NSF under Grant No. IIS-9820657. We thank Jon Wilkenfeld for his comments and Dr. Shifra Hochberg for editorial assistance.

# Contents

# 1 Introduction

In multi-agent environments where agents use resources to satisfy their goals, two agents may need the same resource simultaneously. Sharing a common resource requires a coordinating mechanism that will manage the use of the resource. We extended and applied the strategic model of negotiation [KWZ95] to enable self-interested, rational agents to share resources, when there is no central controller. This strategic-negotiation is a process that may include several iterations of offers and counter offers [OR90, Rub82]. A major consideration in this model has been to reduce overhead costs resulting from planning and negotiation time. In this paper we focus on bilateral negotiation in which one agent uses the resource during the negotiations, while the other agent waits to gain access to the resource.

According to the utilitarian paradigm, a rational agent's interactions with the environment should be guided by the principle of expected utility maximization. We apply this paradigm and develop a formalization of the agent's utility function. Using these utility functions and our negotiation mechanism, agents who need to resolve conflict with respect to resource usage apply simple and stable negotiation strategies that result in efficient agreements on the usage of the resources without any delays. Simulation results show that our mechanism performs as well as a centralized scheduler, is more flexible, and also has the property of balancing the use of resources.

In section 2, we describe the environment which we consider and provide formal definitions of the components of our model. In Section 3, we provide the strategies which are in perfect equilibrium. In Section 4, we present our simulation results, and in Section 6, we summarize the results and suggest further directions for future research.

# 2 Description of the Environment

We consider bilateral negotiation between two self-motivated agents that need the same resource in order to satisfy their goals. Time constraints and a deadline are associated with each goal. One of the agents - the **Attached Agent** (denoted $A$)- uses a resource that another agent - the **Waiting Agent** (denoted $W$)- needs. So, $W$ starts a negotiation process to obtain access to the resource. Possibly, $A$ has performed work before $W$ started the negotiation. During the negotiation process, $A$ continues to hold the resource and to work toward its goal. $W$ cannot move on to another goal and return to this goal later, since the goals are given in a meaningful order.

## 2.1 Goals and resources

An agent needs a resource in order to fulfill one of its goals, which is formally defined in the following definition.[1]

**Definition 1 Goal:**
*A goal is a tuple with six elements* $< g, t_{min}, t_{max}, dl, m, r >$, *where*

- $g$ *is a unique goal identification number. We denote the set of all the goal identification numbers by* $GI$.[2]
- $t_{min}$- *denotes that an agent must work a minimum of at least* $t_{min}$ *time periods on the goal without interruptions in order to receive any payment. That is, this is the minimal amount of time required to partially satisfy the goal.*
- $t_{max}$- *is the maximal number of time periods the agent can work on the goal.*
- $dl$- *specifies the deadline for accomplishing this goal, i.e., the number of time units from the time the agent receives the goal until the goal stops being relevant.*
- $m$ *is the payment the agent receives per time period after a reduction of the fee for using the resource needed for this goal.*
- $r$ *is the resource needed for this goal.*

## 2.2 The negotiation process

We will assume that the goal $A$ is working on during the negotiation is $< g^A, t^A_{min}, t^A_{max}, dl^A, m^A, r^A >$, denoted $g^A$, and the goal that $W$ would like to fulfill (i.e., its reason for the negotiation) is $< g^W, t^W_{min}, t^W_{max}, dl^W, m^W, r^W >$, denoted $g^W$, and $r^A = r^W$.

$W$ starts a negotiation process to obtain access to the resource. Our strategic model of negotiation which the agents use is a model of Alternating Offers. The agents can take actions in the negotiation only at certain times in the set $I\!N = \{0, 1, 2...\}$ that are fixed in advance. These time periods are identical to the time periods referred to in the definition of the goal. We will use $t$ to denote the current negotiation period.

At the first time period, $t = 0$, $W$ makes an offer to $A$. $A$ can accept the offer (choose Yes), leave the negotiation (choose Leave), opt out of the negotiation (choose Opt) or reject the offer (choose No) and make a counter offer. If $A$ accepts the offer, the agreement is implemented immediately. Also, if $A$ Opts out or Leaves, then its action is taken immediately. If $A$ rejects the offer and makes a counter offer, $W$ will respond during the next time period ($t = 1$) by either accepting the offer, leaving the negotiation, opting out or rejecting the

[1] A summary of the notations used in this chapter appears in table 1.
[2] Sometimes, we will refer to the goal using its identification number.

| | Notation | Description | Definition |
|---|---|---|---|
| Goal | $g$ | goal identification | Def. 1 |
| | $t_{min}$ | minimum time periods needed for working in order to get paid for this goal | |
| | $t_{max}$ | maximum time periods needed for working on this goal | |
| | $dl$ | deadline: the number of time units from the goal's arrival time in which the goal is still relevant | |
| | m | payment per time period | |
| | $r$ | the resource needed for this goal | |
| | $done^i$ | periods agent $i$ has been working on the goal so far | |
| | $g^A$ | current goal that $A$ is working on — $< g^A, t^A_{min}, t^A_{max}, dl^A, m^A, r >$ | |
| | $g^W$ | the goal that $W$ wants to work on — $< g^W, t^W_{min}, t^W_{max}, dl^W, m^W, r >$ | |
| Offer | s | the time periods $A$ gets to keep the resource according to an agreement | Def. 2 |
| | n | the time periods $W$ gets to use the resource according to an agreement | |
| $\tilde{s}^{W,t}$ | $< s, n >$ | best agreement for $A$ in time period $t$ that is not worse for $W$ than opting out | Def. 6 |
| | $s^t_A$ | the additional time periods that $A$ needs in time period $t$ in order to completely accomplish its goal | Def. 7 |
| | $O^l$ | the latest offer that was made in the negotiations | Sec 3 |
| General | q | time periods needed for repairing the resource after opting out | Sec 2 |
| | $c$ | cost for $W$ per negotiation period | Sec 2.2 |
| Time periods | $t^W_{neo}$ | the earliest time period in which agent $W$ does not have enough time to perform $t^W_{min}$ before its deadline after Opt | Sec. 3.1 |
| | $t^W_{ne}$ | the earliest time period in which $W$ does not have enough time to perform $t^W_{min}$ before its deadline | |
| | $\hat{t}^A$ | the time in which $A$ would finish working on its goal | |
| | $\hat{t}^W$ | the earliest time in which $W$'s utility from leave is not lower than its utility from any other option | |
| | $\hat{t}$ | the earliest time period between $\hat{t}^W$ and $\hat{t}^A$ | |

Table 1: A short description of all notations used in this paper. The index $i$ refers to either agent $A$ or $W$.

offer and making a counter offer during the next period ($t = 2$), and so on and so forth. Negotiation continues until one of the agents agrees to a given offer, Opts out, or Leaves the negotiation. If one of the agents Leaves, the negotiation ends, and the opponent can obtain the resource. $W$ can also decide not to start the negotiation at all and Leave during the first time period. When one of the agents Opts out, the resource is not available for $q \geq 1$ time periods.[3] We denote by $p_i \in [0,1]$, $i \in \{A, W\}$ the probability that agent $i$ will be able to use the resource as much as it is needed after an agreement is implemented or after the recovery period from Opting out is over.[4]

An offer that is made in the negotiation refers to two aspects of the use of the resource, as shown in the next definition.

**Definition 2 (Agreement:)** *An offer that could become an agreement is a pair $< s, n >$, such that (i) $s$ is the number of steps that $A$ will continue to keep the resource, that is, the number of steps $W$ has to wait, and (ii) $n$ is the number of steps $W$ gets to keep the resource.*

*The set of possible offers is $\mathcal{S} = \{(s,n) \in I\!N^2 : s \geq 0 \;,\; n \geq 0\}$.*

Note that $s$ does not determine $n$ uniquely. Rather, both $s$ and $n$ are subject to negotiation. In particular, in an agreement, $(s, n)$, both $s$ and $n$ are not restricted. However, if $s$ is too large, then $W$ will not have enough time to perform its goal before its deadline. If $n$ is too large, then since holding a resource without using it is costly, it will not be profitable for $W$. Implementing an agreement means that $A$ will continue to work for $s$ time periods and then $W$ will work for $n$. We will see that even after an implementation of an agreement the agents' goals may be only partially satisfied and they may try to obtain additional access to the resource. We demonstrate these RAE environments

**Example 1** *In the first example, NASA has embarked on a scientific mission to Mars that involves sending several mobile robots.[5] The European Space Agency (ESA) has also sent several mobile robots to Mars. Both NASA and ESA's robots work in the same environment. The missions of the robots involve collecting soil samples from different locations and at*

[3] The waiting time may be required for example, if the resource is consequently damaged, when one of the agents opts out.

[4] In order to simplify the agents' utility function, we assume that agent $i$ does not try to estimate whether its opponent will start a second round of negotiations if agent $i$ gets access to the resource after the event. Suppose that $i$ can gain $V_i$ if it will be able to use the resource as much as it is needed after the event. We found out via simulations, that it is possible to determine $p_i$ in such a way that $p_i V_i$ is a good estimation for agent $i$'s expected utility.

[5] Given the 1997 success of NASA's Pathfinder on Mars [GCE$^+$97], and the 1994 success of DANTE II in exploring the crater on the Mt. Spurr volcano in Alaska, it seems that such scenarios may be realistic in the near future (see also [Ber98]). NASA's current plan is to send a pair of wheeled robots to search for evidence of water on Mars in 2003.

*different depths. To satisfy its goal, a robot will need one of the digging tools. The tools were sent to Mars by a third company, which charges NASA and ESA according to their use of the equipment, but does not schedule its usage. When a robot encounters a situation in which it needs a resource that is used by another robot, it may start a negotiation session.*

*In a different example, suppose there are several automated bankers, and each has its own communication system to communicate with its customers and with other branches of the bank. In addition there is a public communication line available for payment. The automated bankers usually use their own communications systems; however, in case they are experiencing an overload, or in case their communications system is down, they use the public domain system. If one of the automated bankers would like to use the public line when another automated banker is already using it, our model is applicable.*

*The same situation may occur when information servers are sharing a public communication line. Similarly, a person that would like to use a public phone, when another person is using it may face a similar problem.*

We assume that each agent has a utility function:
$U^i : \mathcal{S} \cup \{\mathbf{Opt}, \mathbf{Leave^W}, \mathbf{Leave^A}\} \times GI \times I\!\!N \times I\!\!N \rightarrow I\!\!R$. A utility function associates with each of the possible outcomes, the expected benefits of the agent from the outcome. The possible outcomes are an agreement (i.e. a member of $\mathcal{S}$), opting out and Leave by one of the agents. The expected benefits depends on goal identification (i.e., a member of $GI$), the number of periods the agent has been working on the goal until the negotiation has started ($done^i$), and the time period of the negotiation (i.e., a member of $I\!\!N$). When $done^i$ and $g^i$ are clear from the context or do not influence the agent's utility, we will not specify them, i.e., we will write $U^W(Leave^W, t)$ instead of $U^W(Leave^W, g^W, done^W, t)$.

## 2.3 Assumptions

We consider only environments that satisfy the following assumptions:

**Monetary system:** There is a monetary system in the environment which is used for the payments mentioned above, as well as for other costs described below.

**Utility maximization:** The agents try to maximize their payments and minimize their costs.

**Minimal consequence work:** No payments will be made to the agent, unless $t_{min}$ time periods were completed without interruption. The work toward achieving a goal can be stopped and resumed later, on condition the agent has worked for $t_{min}$ periods on it. However, if it is stopped before the agent has worked on it at least $t_{min}$ time periods no payment will be made. The agent will need to start from the beginning if it would like to satisfy the goal.

**Maximal number of periods:** The maximal payment that could be made is $m \cdot t_{max}$ for working on the goal for $t_{max}$ time periods. No payment will be made for additional time periods.

**Cost of holding an unused resource:** An agent that is holding a resource without using it, e.g., its goal's deadline passed, is charged a cost for holding the resource.

**Cost of the negotiations:** $W$ has a fixed cost $c \in \mathbb{R}$ for each negotiation period. The cost is due to communication costs, etc., which it needs to pay.

**Outcome with the same utility:** When two outcomes have the same utility, an agent will prefer the one in which the resource will be free in the future, as much as possible. In particular, when an agent's utility from leaving and from opting out is the same, it will not opt out of the negotiation. Similarly, when an agent's utility from leaving and from accepting an offer is the same, it will not accept the offer but will leave.

In [Sch96] we presented a detailed description of a utility function which can be used in such cases. Here, we only demonstrate the agents' utility functions, using the following example.

**Example 2** We return to the example of the robots on Mars. Suppose that at some given time, a robot sent by NASA, called RobotN, is working on goal G1, which is to dig a hole in Mars' surface using a special digging tool denoted 1002. Its task should take at least 15 minutes and should not last more than 70 minutes (the rock's structure is unknown to the researchers, and therefore a time interval is given instead of an exact amount of time). The deadline of this goal is set at 79 minutes. The payment for each productive minute is 4. We formally specify this goal by $< G1, 15, 70, 79, 4, 1002 >$. Similarly, RobotE is a robot sent by ESA, and one of its goals is to dig a small hole in another location. We specify this goal by $< G2, 10, 20, 30, 4, 1002 >$. That is, $t^W_{min} = 10$, $t^W_{max} = 20$, $W$'s deadline is 30, and its payment, similar to RobotN, for each productive minute is 4. RobotN has started working on its goal first. It has already been working for four minutes ($done^A = 4$) when RobotE wants to start working on its goal and realizes that RobotN is using the resource it needs. This situation requires negotiation, where RobotN is the Attached agent and RobotE is the Waiting agent. Opting out in this case causes the resource to go out of use for eight minutes

($q = 8$). After opting out or after an agreement is implemented (i.e., $A$ has worked for $s$ time periods and $W$ for $n$ time periods), if the agents weren't able to fully satisfy their goals and they still have enough time (i.e., their deadlines haven't arrived yet) they, may try to get access to the resource. We assume that both robots have equal probability of gaining access to the tool and keep on using it in such a case ($p_W = p_A = 0.5$). Since RobotE initiates the negotiation, it has costs of 2 per minute for the communication.

Let us consider the utility of the robots from a specific possible agreement. Suppose the robots agree at the beginning of the negotiation (i.e., $t = 0$) that RobotN will continue working for an additional 20 minutes and will then allow RobotE to use the tool for 10 minutes, i.e., $< 20, 10 >$. This will allow both RobotN and RobotE to perform their minimal required time periods, but will allow RobotN to perform for a few more time periods before the agreement is implemented, and possibly additional time periods after the agreement is implemented. In particular, RobotN, which plays the role of $A$, is allowed to work 20 minutes according to the agreement. Thus its utility from this work is $20 \cdot 4 = 80$. Working for 20 minutes is more than its minimal requirement ($t^A_{min} = 15$). But, even with the 4 minutes RobotN had worked before the negotiation has started ($done^A = 4$), it is still less than its maximal time ($t^A_{max} = 70$). RobotN has enough time to work an additional 20 minutes since its deadline is in 75 minutes (note that the original deadline was 79 minutes from the time it started working on the goal, but it has already worked for 4 minutes).

After the agreement is implemented, i.e., after RobotN has worked for 20 minutes and RobotE has worked for 10 minutes, RobotN will have 45 minutes until its deadline (which together with the minutes worked before the agreement will give it 69 minutes, i.e., almost its maximum). However, the probability that it will be able to work these 45 minutes is only 0.5. Thus its expected utility from working after the agreement is implemented is $0.5 \cdot 45 \cdot 4 = 90$. Summing these utilities we obtain that $U^A(< 20, 10 >, G1, 4, 0) = 170$.

The only utility for RobotE, which plays the role of $W$, is attained from reaching the agreement. It does not have future time periods since its deadline will arrive after the agreement is implemented, i.e., in 20+10 minutes. Thus its payments are $10 \cdot 4 = 40$. Since the agreement is reached during the first time period of the negotiation, it does not have any additional costs. Thus, $U^W(< 20, 10 >, G2, 0, 0) = 40$.

Opting out at the beginning of the negotiation ($t = 0$) is worse for RobotN than the agreement $< 20, 10 >$. If opting out occurs, RobotN won't have any productive time periods before it. However, it may be able to perform most of its maximal required time, i.e., 67 minutes, after the resource becomes available again and before its deadline arrives. The probability that it will be able to work these 67 minutes is only 0.5. Thus, $U^A(Opt, G1, 4, 0) = 0.5 \cdot 67 \cdot 4 = 134$.

RobotE, which plays the role of $W$, will have the chance to work its total maximal time after opting out, but again only with a probability of 0.5. Thus $U^W(Opt, G2, 0, 0) = 0.5 \cdot 20 \cdot 4 = 40$. This is equal to the utility from the agreement $< 20, 10 >$, where it works for 10 minutes with certainty.

## 2.4 Subgame perfect equilibria

We are now ready to consider the problem of how a rational agent will choose its negotiation strategy. A useful notion is the Nash Equilibrium [Nas53, LR57]. It uses the concept of a strategy profile that is an ordered set of strategies, one for each player.

**Definition 3 (Nash Equilibrium)** *A strategy profile $(f_1, f_2)$ is a* Nash equilibrium *of a model of alternating offers, if each agent $i \in \{1, 2\}$ does not have a different strategy yielding an outcome that it prefers to that generated when it chooses $f_i$, given that the other agent $j$ chooses $f_j$.*

This means, that if both agents use the strategies specified for them in the strategy profile of the Nash equilibrium, then no agent will benefit by deviating and using another strategy. However, the use of Nash equilibrium in a model of alternating-offers leads to an absurd Nash equilibria [Tir88]: an agent may use a threat that would not be carried out if the agent were put in the position to do so, since the threat move would give the agent lower payoff than it would get by not taking the threatened action. This is because Nash equilibrium strategies may be in equilibrium only in the beginning of the negotiation, but may be unstable in intermediate stages. In the following definition, the concept of subgame perfect equilibrium (SPE) [OR94] is defined, which is a stronger concept, and will be used in order to analyze the negotiation.

**Definition 4 (Subgame perfect equilibrium:)** *A strategy profile is a* subgame perfect equilibrium *of a model of alternating offers if the strategy profile induced in every subgame is a Nash equilibrium of that subgame.*

This means that at any step of the negotiation process, no matter what the history is, no agent will benefit by deviating and using any strategy other than that defined in the strategy profile.

## 2.5 Acceptable agreements

The main question in identifying strategies that are in subgame perfect equilibrium is what are the offers that may be acceptable to the agents. It is clear that such offers must be better than (or equal to) opting out or leaving for both agents. Otherwise, they will prefer the other options to reaching an agreement.

**Definition 5 (Agreements that are not Worse than Opting Out)** *For every period* $t \in \mathbb{N}$ *and agent* $i \in \{W, A\}$ *let* $Possible_i^t \stackrel{\text{def}}{=} \{s^t \mid s^t \in S,\ U^i((s^t, t)) \geq U^i((\mathbf{Opt}, t))\}$ *be the set of all the possible agreements that are not worse for agent* $i$ *in period* $t$ *than opting out in period* $t$. $Possible^t$ *is the set of agreements which are preferred by all the agents over opting out, i.e.,* $Possible^t = Possible_W^t \cap Possible_A^t$.

Note that Possible$^t$ is never empty. The agreement $< 0, 0 >$ is always not worse to both agents than opting out since $< 0, 0 >$ allows both agents the possibility of obtaining the resource again immediately. This is better than trying to gain access to the resource after $q$ time periods, as in opting out.

Even though it is clear that the agreement that will be reached belongs to Possible$^t$, we still have to find out which of the members of Possible$^t$ will be reached when the agents follow their subgame perfect-equilibrium strategies. Since $A$ is using the resource during the negotiation and does not have negotiation costs, it has an advantage over $W$. Thus, as we will prove later, there is a subgame-perfect equilibrium in which the best agreement for $A$ in Possible$^t$ will be the basis of the strategies.[6]

**Definition 6 (Acceptable offer:)** $\tilde{s}^{A,t} =< s, n >$ *is the best agreement for* $A$*, at time period* $t$*, which is still not worse for* $W$ *than opting out. That is,* $U^i((\tilde{s}^{i,t}, t)) = max_{s \in Possible^t} U^i((s, t))$.

*If there are several such agreements, the one in which the resource is left free for the longest time is chosen.*

The next definition deals with $A$'s needs: how many additional time periods it needs in period $t$ in order to completely accomplish its goal, given the specifications of its goal and the current time period. At a given time period $t$ of the negotiations, $A$ has already worked for $t + done^A$ time periods. In the best case, it will be able to work for enough periods to reach its maximal requirements, i.e., $t_{max}^A - done^A - t$. However, its deadline may arrive before this, and thus it will be able to work only for the time left until its deadline, i.e., $dl^A - done^A - t$. This is summarized in the following definition.

**Definition 7** $s_A^t$**:**

$$s_A^t = min\{dl^A - done^A - t, t_{max}^A - done^A - t\}.$$

[6] Whether this subgame-perfect equilibrium is unique is an open question.

When $A$ works for additional $s_A^t$ time periods, after $t$ time periods of negotiation, it will accomplish the feasible part of its goal. Therefore, it is easy to prove the following lemma.[7]

**Lemma 1 ($< s_A^t, 0 >$ is the best agreement for $A$.)**

$$\forall t, n, s' \neq s_A^t, U^A(< s', n >, g^A, done^A, t) \leq U^A(< s_A^t, 0 >, g^A, done^A, t)$$

∎

[Sch96] identified the values of $\tilde{s}^{A,t}$ given the agents' utility functions. Here we simply demonstrate these findings using an example and present our findings concerning the agents' preferences.

**Example 3** We return to the example of the robots on Mars (Example 2) and compute $\tilde{s}^{A,0}$. Recall that RobotN's goal is $< G1, 15, 70, 79, 4, 1002 >$, i.e., $t^A_{min} = 15$, $t^A_{max} = 70$, and the deadline is 79. The goal of RobotE is $< G2, 10, 20, 30, 4, 1002 >$, i.e., $t^W_{min} = 10$, $t^W_{max} = 20$, and the deadline is 30. In this case, RobotE, which plays the role of $W$, has enough time to work for at least its minimal required time ($t^W_{min} = 10$) after opting out, i.e., $dl^W - q \geq t^W_{min}$, where $dl^W = 30$ and $q = 8$. Furthermore, $W$ can accomplish its goal completely, by working on it $t^W_{max}$ time periods after opting out since $W$ has more than $t^W_{max} = 20$ time periods after opting out.

RobotN plays the role of $A$. At the beginning of the negotiations, the number of minutes left for RobotN to work in order to fully accomplish its goal is 66. This is because the goal requires 70 minutes and it has already worked for 4 minutes. The 66 minutes are productive since it has enough time until its deadline. That is, $s_A^0 = 66$. However, if RobotN continues working for 66 minutes, RobotE will not be able to even partially accomplish its goal since its deadline is 30 minutes. RobotE should consider two possible agreements, $< 20, 10 >$ and $< 0, 0 >$, which are better for RobotE than opting out. In $< 20, 10 >$, $A$ will work until the last minute, after which RobotE will not be able to work its minimal required time periods. In $< 0, 0 >$, $A$ will give up the resource and has a probability of 0.5 of gaining it back immediately and use it as much as it is needed. As calculated in Example 2, $U^A(< 20, 10 >, G1, 4, 0) = 170$. It is easy to see that $U^A(< 0, 0 >, G1, 4, 0) = 140$. This means that RobotN's utility from continuing to work for 20 minutes, then giving the tool to RobotE for 10 minutes and waiting, and only after it finishes try to gain access to the tool is higher. This option is better than leaving the resource immediately and having a chance equal to that of RobotE to gain the tool, and to work possibly for 70 minutes. This is the

[7] The formal proofs of all the lemmas and the theorem of this paper can be found in [Sch96]. Here, we only present the logic behind the proofs, when the proofs are not trivial.

case, even though in the first possibility, i.e., $< 20, 10 >$, it will not be able to satisfy its maximal requirements. Thus $\tilde{s}^{A,0} =< 20, 10 >$.

It is useful to observe how $\tilde{s}^{A,t}$ changes over time. At periods 1 and 2, the conditions do not change significantly, and thus $\tilde{s}^{A,1} =< 19, 10 >$ and $\tilde{s}^{A,2} =< 18, 10 >$. At time 3, RobotE does not have enough time before its deadline to do $t^W_{max} = 20$ after opting out, but still $\tilde{s}^{A,3} =< 17, 10 >$, allowing RobotE to work for its minimal requirement before its deadline. The situation changes only at time 13, when RobotE does not have enough time to work for its minimal requirement before its deadline after opting out. In this case, $\tilde{s}^{A,t} =< s^t_A, 0 >$, i.e., $\tilde{s}^{A,13} =< 53, 0 >$ since $A$ has worked for 4 minutes before the negotiations started ($done^A = 4$) and it continues to work for 13 minutes during the negotiations. Note that RobotN's utility from $\tilde{s}^{A,t}$ does not change over time until $t = 13$. However, $\tilde{s}^{A,13}$ at period 13 ($U^A(< 53, 0 >, G1, 4, 13) = 66 * 4 = 264$) is better than $\tilde{s}^{A,0}$ at the first time period ($U^A(< 20, 10 >, G1, 4, 0) = 170$).

Robot E's utility from $\tilde{s}^{A,t}$ is reduced over time. In particular, $U^W(< 20, 10 >, G1, 4, 0) = 40$; since RobotE pays 2 per minute for negotiation costs, $U^W(< 19, 10 >, G1, 4, 1) = 40-2 = 38$ and $U^W(< 18, 10 >, G1, 4, 2) = 36$. Its utility from $\tilde{s}^{A,13}$ is really low, since it does not gain any productive working time, but it needs to pay for the negotiation i.e., $U^W(< 53, 0 >, G1, 4, 13) = -26$.

## 2.6 The agents' preferences in respect to different outcomes

An important question that we consider before specifying the strategies for $A$ and $W$, which are in equilibrium, are the preferences of the agents between opting out and $\tilde{s}^{A,t}$, and the way they change over time. In [Sch96], a detailed discussion and formal results are presented concerning this issue. Here we only summarize the most interesting results, and present a lemma that is needed for the proofs in the next section.

### 2.6.1 $W$'s preferences

In general, $W$ loses over time. Thus we assume that the following properties are true concerning its preferences.

**$W$'s utility from opting out now is higher than from opting out later:** If $W$ Opts out now, it has the option of more future time periods to accomplish its goal and thus its expected utility is higher. In addition, if it spends less time negotiating because of its opting out earlier, its negotiation costs are lower.

**$W$'s utility from $\tilde{s}^{A,t}$ now is higher than from $\tilde{s}^{A,t+1}$ at $t+1$:** $W$'s share in $\tilde{s}^{A,t}$ does not increase over time. In addition, it pays for the negotiation.

**$W$'s utility from $\tilde{s}^{A,t}$ now is higher than its utility from opting out later:** This is the result of $W$'s gaining higher utility from opting out sooner than from opting out later and that $\tilde{s}^{A,t}$ is always not worse than opting out at time $t$.

The following lemma indicates that when $W$'s utility from opting out and Leave is the same or its utility from Leave is higher than its utility from opting out, then it remains this way during the negotiation. This lemma is used in the next section, and its proof follows from the definition of $W$'s utility for Opt and Leave.

**Lemma 2 ($Leave^W$ vs. Opt.)**
$\forall t, t' \in I\!N$, if $U^W(Leave^W, g^W, done^W, t) \geq U^W(Opt, g^W, done^W, t)$, then
if $t' > t$, $U^W(Leave^W, g^W, done^W, t') \geq U^W(Opt, g^W, done^W, t')$. ∎

### 2.6.2 $A$'s preferences

We assume that the following properties are true concerning its utility function.

**Opting out now vs. opting out later:** In most of the cases $A$'s utility from opting out at period $t+1$ is higher than if it opts out at period $t$, since it is the one that holds the resource and is being paid for its work. However, there are periods of time in which this is not correct: when $A$ has not yet performed $t^A_{min}$ periods, it does not get paid for each negotiation period, but needs to pay for holding the resource, and therefore its utility from opting out at a given time period $t$ is higher than from opting out at $t+1$. In this way $A$ saves the payment for holding its resource an additional time period. Another case in which $A$'s utility is higher if the negotiation terminates as soon as possible is after $A$ has completed its maximum periods. Any additional negotiations cause it to over-pay for holding the resource when it doesn't actually need it and it is not paid at all.

**$A$ obtains the same utility from $\tilde{s}^{A,t} =< s^t_A, n >$ at $t$ and $\tilde{s}^{A,t+1} =< s^{t+1}_A, n >$ at $t+1$:** When $\tilde{s}^{A,t} =< s^t_A, n >$ for some $n \in I\!N$ and $s^t_A > 0$, $A$ gains its maximal utility.[8] As $t$ increases, $s^t_A$ decreases, respectively. Therefore, instead of $\tilde{s}^{A,t} =< s^t_A, n >$, we have $\tilde{s}^{A,t+1} =< s^t_A - 1, n >$. But since $A$ is working on its goal during the negotiation, its utility remains the same.

[8] $s^t_A$ is the maximum number of time periods needed by $A$ at period $t$ in order to accomplish the feasible part of its goal.

**$A$'s utility from $\tilde{s}^{A,t} =< 0,0 >$ at $t$ is higher than from $\tilde{s}^{A,t+1} =< 0,0 >$ at $t+1$:** $\tilde{s}^{A,t}$ is equal to $< 0,0 >$ when $A$ does not benefit from holding the resource, either because it hasn't reached $t^A_{min}$ yet, or because it has accomplished its goal: i.e., it has worked more than $t^A_{max}$ but is paying for holding the resource. In both cases, $A$'s utility from $< 0,0 >$ at $t$ is higher than its utility from $< 0,0 >$ at $t+1$. Note, however, that in some cases the utility from $< 0,0 >$ at period $t$ may be equal to the utility from leaving at $t$.

# 3 Subgame perfect equilibrium strategies

Having discussed the agents' utility from different outcomes of the negotiation, we are ready to identify the strategies that are in subgame perfect equilibrium. The details of the strategies that are in SPE depend on the agents' utility from $\tilde{s}^{A,t}$, opting out, and Leaving. We will present the strategies that are in subgame perfect equilibrium and show that when the strategies of the subgame perfect equilibrium are used, the negotiation ends, at most, after two negotiation time periods.

## 3.1 Time periods when the negotiation ends

Before discussing the strategies that are in equilibrium, we identify the time periods in which it is clear that the negotiation will end.

**Lemma 3 ($A$'s strategy when $s^t_A = 0$.)** When $s^t_A = 0$, if it is $A$'s turn to respond to an offer and the negotiation has not ended at $t-1$, then $A$ will Leave the negotiation. ∎

In the proofs of the rest of the lemmas and theorems, we will concentrate only on time periods $t$, in which $s^t_A > 0$, since from the previous lemma it is clear that the negotiation will not continue after that point. The first time period in which $s^t_A = 0$ is denoted $\hat{t}^A$. In addition there are situations where $W$ will always Leave the negotiation. This is when its utility from Leave is not lower than its utility from $\tilde{s}^{A,t}$.

**Lemma 4 (Strategies when $Leave^W$ is better for $W$ than $\tilde{s}^{A,t}$.)**
Suppose $U^W(Leave^W, g^W, done^W, t) \geq U^W(\tilde{s}^{A,t}, g^W, done^W, t)$. If it is $W$'s turn to respond to an offer and the negotiation has not ended at $t-1$, then $W$ will Leave.

If it is $A$'s turn to respond to an offer, $s^t_A > 0$, and the negotiation has not ended at $t-1$, then $A$ will say No and offer $\tilde{s}^{A,t+1}$. ∎

To simplify the notation, we define $t^W_{ne} = dl^W - t^W_{min} + 1$, and $t^W_{neo} = dl^W - t^W_{min} - q + 1$. Intuitively, these two points in time indicate changes in $W$'s utility for different actions: when reaching $t^W_{neo}$, $W$ gains the same utility from both opting out and Leave, since, from that point on, it will not have enough time periods to work $t^W_{min}$ time periods after waiting $q$ time periods. When reaching $t^W_{ne}$, $W$ has no preference between Leaving and accepting $\tilde{s}^{A,t}$, too, since it will not have enough time to work $t^W_{min}$ time periods if $\tilde{s}^{A,t}$ is implemented. In some situations, $W$ will become indifferent to $\tilde{s}^{A,t}$ as Leave for some $t^W_{neo} \leq n < t^W_{ne}$. We denote the earliest time period in which this happens by $\hat{t}^W$. For example, suppose $dl^W = 14$, $q = 6$ and $t^W_{min} = 5$. In this case, $t^W_{neo} = 14 - 5 - 6 + 1 = 4$ and $t^W_{ne} = 14 - 5 + 1 = 10$. It is easy to see that for every $t \geq t^W_{neo}$, $\tilde{s}^{A,t} =< s^t_A, 0 >$ [Sch96]. Suppose that at time $t^W_{neo} = 4$, $s^4_A = 3$. Then $W$'s utility from $\tilde{s}^{A,t}$ at that time is higher than its utility from Leave, since according to the agreement, after $A$ leaves the resource, $W$ will still have 7 time periods to try to work on its goal. Thus, $\hat{t}^W = 10$. However, if, for example, $s^4_A = 8$, Leave is better for $W$ at period 4 than $\tilde{s}^{A,4}$, thus $\hat{t}^W = 4$. Note, however, that in the first case, where $s^4_A = 3$, $A$ will Leave the negotiation before the earliest time in which $W$ prefers to Leave ($\hat{t}^W = 10$), i.e., the time in which $A$ would finish working, $\hat{t}^A$, is earlier than $\hat{t}^W$.

## 3.2 Time periods near the end of the negotiation

From Lemmas 3 and 4 it is clear that the negotiation will end at the earliest time between the earliest time in which $W$'s utility from Leave is not lower than its utility from any other option ($\hat{t}^W$) and the time in which $A$ would finish working ($\hat{t}^A$) if it hasn't ended before so far. We will try to construct the strategies of the agents that are in subgame perfect equilibrium backward from these points.

### 3.2.1 $W$ prefers Leave than $\tilde{s}^{A,t}$ in a time period before $s^t_A$ becomes 0 ($\hat{t}^W < \hat{t}^A$)

We consider the case in which the earliest time in which $W$ prefers to Leave is earlier than the time in which $A$ would finish working, i.e., $\hat{t}^W < \hat{t}^A$. We first specify the strategies for $A$ and $W$ at $\hat{t}^W - 1$.

**Lemma 5 (Strategies at $\hat{t}^W - 1$.)** Suppose the negotiation hasn't ended at $\hat{t}^W - 2$.

**$W$'s strategy:** If at time $\hat{t}^W - 1$ it is $W$'s turn to respond to an offer, then it will use the following strategy:

1. If $U^W(Leave^W, \hat{t}^W - 1) < U^W(Opt, \hat{t}^W - 1)$, then if $U^W(Opt, \hat{t}^W - 1) \leq U^W(O^l, \hat{t}^W - 1)$, then say Yes. Otherwise, Opt.

2. Otherwise, if $U^W(Leave^W, \hat{t}^W - 1) \geq U^W(Opt, \hat{t}^W - 1)$, then if $U^W(Leave^W, \hat{t}^W - 1) < U^W(O^l, \hat{t}^W - 1)$, then say Yes. Otherwise, Leave.

**$A$'s strategy:** If at time $\hat{t}^W - 1$, it is $A$'s turn to respond to an offer, then it says No and offers $\tilde{s}^{A,t+1}$.

▮

The reasoning behind the above lemma is as follows. In $W$'s strategy, step 1 considers the case where Opt is better for $W$ than Leave. In this case, $W$ will compare between Opt and accepting the Offer made by $A$ (i.e., $O^l$) and choose the best option for itself. In the second step, Leave is better than Opt, and thus $W$ will choose between Leaving and accepting $A$'s offer.

$A$'s strategy above is very simple. It would like $W$ to Leave in the next time period, and thus rejects $W$'s current offer knowing that $W$ will Leave in the next time period.

Next, we consider the agents' behavior at time $\hat{t}^W - 2$. If it is $W$'s turn to respond to an offer at $\hat{t}^W - 2$, then it will try to end the negotiation at that time period. If the negotiation continues to the next time period, $A$ will make $W$ Leave at $\hat{t}^W$. Since $W$ loses over time, it will prefer reaching an agreement, Leaving or Opting out in $\hat{t}^W - 2$. If it is $A$'s turn to respond to an offer, it will try to make $W$ Leave the negotiation. If it is not possible to make $W$ Leave (since Opt is better for $W$ than Leave), $A$ will try to prevent $W$ from opting out by either accepting its offer or by offering $\tilde{s}^{A,t+1}$, which is not worse for $W$ than opting out. This is stated formally in the next lemma.

**Lemma 6 (Strategies at $\hat{t}^W - 2$.)** Suppose the negotiation hasn't ended at $\hat{t}^W - 3$, and $t = \hat{t}^W - 2$.

**$W$'s strategy:** If it is $W$'s turn to respond to an offer, then it will use the following strategy:

1. If $U^W(Leave^W, t) < U^W(Opt, t)$, then if $U^W(Opt, t) \leq U^W(O^l, t)$, then say Yes. Otherwise, Opt.
2. Otherwise, if $U^W(Leave^W, t) \geq U^W(Opt, t)$, then if $U^W(Leave^W, t) < U^W(O^l, t)$, then say Yes. Otherwise, Leave.

**$A$'s strategy:** If $t = \hat{t}^W - 2$ and it is $A$'s turn to respond to an offer, then it will use the following strategy:

1. If $U^W(Opt, t+1) \leq U^W(Leave^W, t+1)$, then say No and offer $< s^t_A, 0 >$.
2. Otherwise, if $U^A(Leave^A, t) \geq U^A(Opt, t) \geq U^A(O^l, t)$ and $U^A(Leave^A, t) \geq U^A(\tilde{s}^{A,t+1}, t+1)$, then Leave.

3. Otherwise, if $U^A(O^l, t) < U^A(\tilde{s}^{A,t+1}, t+1)$ and $U^A(Opt, t) \leq U^A(\tilde{s}^{A,t+1}, t+1)$, then say No and offer $\tilde{s}^{A,t+1}$.
4. Otherwise, if $U^A(O^l, t) = U^A(\tilde{s}^{A,t+1}, t+1)$, then
   (a) If $O^l = \tilde{s}^{A,t}$, then say Yes,
   (b) Otherwise, say No and offer $\tilde{s}^{A,t+1}$.
5. Otherwise, if $U^A(O^l, t) > U^A(\tilde{s}^{A,t+1}, t+1)$ and $U^A(O^l, t) \geq U^A(Opt, t)$, then say Yes.
6. Otherwise, Opt.

▮

In the lemma above (Lemma 6), $W$'s strategy is simpler than $A$'s (while in the previous lemma, $A$'s strategy was simpler). $W$'s strategy is exactly as in period $\hat{t}^W - 1$ (Lemma 5): it chooses the best option between Opt, Leave, and accepting $A$'s offer. This is because if the negotiation continues to $\hat{t}^W - 1$, it will be $A$'s turn and $A$ will delay the negotiation and make $W$ Leave, as in $\hat{t}^W$. This will yield $W$ a lower utility than taking an action in $\hat{t}^W - 2$.

In the first step of $A$'s strategy above, the case where $W$ prefers to Leave is considered. In this case, $A$ would like to delay the negotiation to the next time period, where $W$ will Leave.

In the second step $A$ compares its utility from $Leave^A$ now and its utility from opting out now, accepting $W$'s current offer and $\tilde{s}^{A,t+1}$ at $t+1$. Since these are the options with the highest utility, if its utility from $Leave^A$ is not lower than its utility from all these options, it will Leave the negotiations.

In step 3, $A$ compares between its utility from accepting $W$'s current offer, its expected utility in the next time period (i.e., its utility from $\tilde{s}^{A,t+1}$ at $t+1$) and its expected utility from Opting out now. If the future option is the best, it will not accept the offer, and wait for $W$ to accept its offer $\tilde{s}^{A,t+1}$ in the next time period.

Step 4 considers the case in which $W$'s offer is as good as the future option of $\tilde{s}^{A,t+1}$. To make $W$ offer $\tilde{s}^{A,t}$ and not an offer that will keep the resource busy, it will accept the offer only when it is equal to $\tilde{s}^{A,t}$.

Step 5 considers the case in which $W$'s offer is better than the future option and Opt. Thus, the best option for $A$ is to accept the offer.

If none of the conditions of steps 1–5 holds, $A$ would conclude that Opt out is the best option, as indicated in step 6.

### 3.2.2 $s_A^t$ becomes 0 before the first period in which $W$ prefers $Leave^W$ to $\tilde{s}^{A,t}$ ($\hat{t}^A < \hat{t}^W$)

We now consider the case where the time in which $A$ would finish working is earlier than the earliest time in which $W$ prefers to Leave ($\hat{t}^A < \hat{t}^W$). We know from Lemma 3 that if it is $A$'s turn to respond to an offer at $\hat{t}^A$, it will Leave. In the next lemma, we consider the case where $t = \hat{t}^A$, but it is $W$'s turn to respond.

**Lemma 7 ($W$'s strategy at $\hat{t}^A$)** Suppose the negotiation hasn't ended at $\hat{t}^A - 1$. If at $t = \hat{t}^A$ it is $W$'s turn to respond to an offer $O^l$, then it will use the following strategy:

1. If $U^W(O^l, t) \geq U^W(Opt, t)$ and $U^W(O^l, t) \geq U^W(Leave^A, t+1)$, then say Yes.
2. Otherwise, if $U^W(Opt, t) > U^W(Leave^A, t+1)$, then Opt.
3. Otherwise, say No and offer $\tilde{s}^{A,t+1}$.

∎

In $W$'s strategy of the lemma above, $W$ compares $Leave^A$ in the next time period, $A$'s offer, and Opt. If $A$'s offer is the best (step 1), then it will accept the offer. If Opt is the best option (step 2), then $W$ will opt out. Otherwise, (step 3) it will wait until the next period by saying No and making a counteroffer.

The next time period to be considered in our backward induction is $\hat{t}^A - 1$.

**Lemma 8 (Strategies at $\hat{t}^A - 1$)** Suppose the negotiation hasn't ended at $\hat{t}^A - 2$.

**$W$'s strategy:** If at time $\hat{t}^A - 1$ it is $W$'s turn to respond to an offer $O^l$, then it will use the following strategy:

1. **Opt is better for $W$ than $Leave^W$:**
   If $U^W(Leave^W, \hat{t}^A - 1) < U^W(Opt, \hat{t}^A - 1)$, then
   (a) If $U^W(Leave^A, \hat{t}^A) \geq U^W(Opt, \hat{t}^A - 1)$, then
      i. If $U^W(O^l, \hat{t}^A - 1) \geq U^W(Leave^A, \hat{t}^A)$, then say Yes.
      ii. Otherwise, say No and suggest $\tilde{s}^{A,\hat{t}^A}$.
   (b) Otherwise, if $U^W(O^l, \hat{t}^A - 1) \geq U^W(Opt, \hat{t}^A - 1)$, then say Yes.
   (c) Otherwise, Opt.
2. **$W$ prefers to Leave now than $A$'s Leaving in the next time period:**
   Otherwise, if $U^W(Leave^W, \hat{t}^A - 1) \geq U^W(Leave^A, \hat{t}^A)$, then

(a) If $U^W(Leave^W, \hat{t}^A - 1) < U^W(O^l, \hat{t}^A - 1)$, then say Yes.

(b) Otherwise, Leave.

3. **Accepting $A$'s offer is the best option for $W$:**

   Otherwise, if $U^W(O^l, \hat{t}^A - 1) > U^W(Leave^A, \hat{t}^A)$, then say Yes.

4. **Waiting for $A$ to Leave is the best option for $W$:**

   Otherwise, say No and offer $\tilde{s}^{A,\hat{t}^A}$.

**$A$'s strategy:** If in time $\hat{t}^A - 1$ it is $A$'s turn to respond to an offer, then it will use the following strategy:

1. **Accepting $W$'s offer is $A$'s best option:**

   If $[U^A(O^l, \hat{t}^A - 1) = U^A(\tilde{s}^{A,\hat{t}^A}, \hat{t}^A)$ and $O^l = \tilde{s}^{A,\hat{t}^A - 1}]$ OR $U^A(O^l, \hat{t}^A - 1) > U^A(\tilde{s}^{A,\hat{t}^A}, \hat{t}^A)$, then say Yes.

2. **$\tilde{s}^{A,\hat{t}^A}$ in the next time period is the best option:**

   Otherwise, say No and suggest $\tilde{s}^{A,\hat{t}^A}$.

∎

In the above lemma, if it is $W$'s turn to respond to $A$'s offer at $\hat{t}^A - 1$, it will choose between $Leave^A$ in the next time period, Opt now, $Leave^W$ now, or accept $A$'s offer. Step 1 considers the case where Opt now is better for $W$ than $Leave^W$. In step 1a, $Leave^A$ in the next time period is better for $W$ than Opt. Thus, $W$ compares $A$'s offer and $Leave^A$. In step 1b, the case where Opt is better than $Leave^A$ is considered, and thus $W$ compares Opt and $A$'s offer.

The rest of the steps consider the case in which $W$ prefers to Leave now rather than to opt out. Step 2 considers the rare case where $W$ prefers to Leave now rather than to wait for $A$ to leave during the next time period. Thus it compares accepting $A$'s offer and $Leave^W$.

In step 3, $A$'s offer is better than $Leave^A$ in the next time period, which is better than $Leave^W$ now, which, in turn, is better than Opt.

The case where $Leave^A$ in the next time period is the best option is considered in step 4. That is, $U^W(Leave^W, \hat{t}^A - 1) \geq U^W(Opt, \hat{t}^A - 1)$, but $U^W(Leave^W, \hat{t}^A - 1) < U^W(Leave^A, \hat{t}^A)$. This may occur when $t^W_{neo} < \hat{t}^A < t^W_{ne}$.

$A$'s strategy in the lemma above, when it needs to respond to $W$'s offer at $\hat{t}^A - 1$ is simpler. First, it is easy to see that $\tilde{s}^{A,\hat{t}^A} =< 0, 0 >$ and that if it is offered by $A$ at $\hat{t}^A$, it will be accepted by $W$. Thus $A$ will compare $W$'s offer and $\tilde{s}^{A,\hat{t}^A}$ in the next time period. Note that if its utility from $\tilde{s}^{A,\hat{t}^A}$ and $O^l$ is the same, it will accept the offer only if $O^l = \tilde{s}^{A,\hat{t}^A-1}$ in order to keep the resource free as long as possible.

We will continue by considering the strategies at $\hat{t}^A - 2$. We will specify the strategies of $\hat{t}^A - 2$ as generally as possible, in order to be able to go on to extend it to earlier time periods.

**Lemma 9 (Strategies at $\hat{t}^A - 2$)** Suppose the negotiation hasn't ended by $\hat{t}^A - 3$.

**$W$'s strategy:** If at time $t = \hat{t}^A - 2$ it is $W$'s turn to respond to an offer, it will use the following strategy:

1. **$A$ prefers $\tilde{s}^{A,t+1}$ at $t+1$ than $\tilde{s}^{A,t+2}$ at $t+2$:**
   If $U^A(\tilde{s}^{A,t+1}, t+1) \geq U^A(\tilde{s}^{A,t+2}, t+2)$, then
   (a) If $U^W(O^l, t) \geq U^W(\tilde{s}^{A,t+1}, t+1)$ and $U^W(O^l, t) \geq U^W(Opt, t)$, then say Yes.
   (b) Otherwise, if $U^W(Leave^W, t) \geq U^W(Opt, t)$, then
      i. If $U^W(Leave^W, t) \geq U^W(\tilde{s}^{A,t+1}, t+1)$, then Leave.
      ii. Otherwise, say No and suggest $\tilde{s}^{A,t+1}$.
   (c) Otherwise, if $U^W(\tilde{s}^{A,t+1}, t+1) \geq U^W(Opt, t)$, then say No and offer $\tilde{s}^{A,t+1}$.
   (d) Otherwise, Opt.
2. **$Leave^W$ is not worse for $W$ than Opt:**
   Otherwise, if $U^W(Leave^W, t) \geq U^W(Opt, t)$, then
   (a) If $U^W(O^l, t) \geq U^W(\tilde{s}^{A,t+2}, t+2)$ and $U^W(O^l, t) > U^W(Leave^W, t)$, then say Yes.
   (b) Otherwise, if $U^W(Leave^W, t) \geq U^W(\tilde{s}^{A,t+2}, t+2)$, then Leave.
   (c) Otherwise, say No and offer $\tilde{s}^{A,t+1}$.
3. **Accepting $A$'s offer now is the best option:**
   Otherwise, if $U^W(O^l, t) \geq U^W(\tilde{s}^{A,t+2}, t+2)$ and $U^W(O^l, t) \geq U^W(Opt, t)$, then say Yes.
4. **Waiting for $\tilde{s}^{A,\hat{t}^A}$ at $\hat{t}^A$ is the best option:**
   Otherwise, if $U^W(\tilde{s}^{A,t+2}, t+2) \geq U^W(Opt, t)$, then say No and offer $\tilde{s}^{A,t+1}$.
5. **Opt is the best option:**
   Otherwise, Opt.

**$A$'s strategy:** If in time $t = \hat{t}^A - 2$ it is $A$'s turn to respond to an offer, then it will use the following strategy:

1. $Leave^W$ **is not worse for** $W$ **than Opt in the next time period:**

   If $U^W(Opt, t+1) \leq U^W(Leave^W, t+1)$, then say No and offer $< s_A^t, 0 >$.

2. $Leave^A$ **is the best option:**

   Otherwise, if $U^A(Leave^A, t) \geq U^A(Opt, t) \geq U^A(O^l, t)$ and $U^A(Leave^A, t) \geq U^A(\tilde{s}^{A,t+1}, t+1)$, then Leave.

3. **Accepting** $W$**'s offer is the best option:**

   Otherwise, if $U^A(O^l, t) < U^A(\tilde{s}^{A,t+1}, t+1)$ and $U^A(Opt, t) \leq U^A(\tilde{s}^{A,t+1}, t+1)$, then say No and offer $\tilde{s}^{A,t+1}$.

4. $A$ **has no preference between** $W$**'s offer now and** $\tilde{s}^{A,t+1}$ **in the next time period:**

   Otherwise, if $U^A(O^l, t) = U^A(\tilde{s}^{A,t+1}, t+1)$, then

   (a) If $O^l = \tilde{s}^{A,t}$, then say Yes,

   (b) Otherwise, say No and offer $\tilde{s}^{A,t+1}$.

5. **Accepting** $W$**'s offer is the best option:**

   Otherwise, if $U^A(O^l, t) > U^A(\tilde{s}^{A,t+1}, t+1)$ and $U^A(O^l, t) \geq U^A(Opt, t)$, then say Yes.

6. **Opting out is the best option:**

   Otherwise, Opt.

▮

In the first step of $W$'s strategy of the lemma above, the case where $A$'s utility from $\tilde{s}^{A,t+1}$ at $t+1$ is higher than its utility from $\tilde{s}^{A,t+2}$ at $t+2$ is considered. In this case, if $W$ offers $A$ $\tilde{s}^{A,t+1}$ in the next time period ($t = \hat{t}^A - 1$), it will be accepted. Thus $W$ will compare accepting $A$'s offer, $O^l$, Opt, and $Leave^W$. In step 1a, accepting $A$'s offer is the best option, and thus $W$ says Yes. In step 1bi, $Leave^W$ is the best option, and in steps 1bii and 1c, $\tilde{s}^{A,t+1}$ is the best option.

The situations in which $A$'s utility from $\tilde{s}^{A,t+1}$ at $t+1$ is not higher that its utility from $\tilde{s}^{A,t+2}$ at $t+2$ are considered in steps 2-5 of $W$'s strategy above. In Step 2, $Leave^W$ is not worse for $W$ than Opt. In this case, $W$ cannot threaten $A$ with opting out, and thus $A$ will try to delay the negotiations until $t+2 = \hat{t}^A$. Thus $W$ will compare $O^l$ (step 2a), Leaving now (step 2b), or $\tilde{s}^{A,t+2}$ at $\hat{t}^A = n+2$ (step 2c).

In steps 3, 4, and 5, the situations where Opt is better for $W$ than $Leave^W$ are considered. $W$ will choose between accepting $A$'s offer (step 3), waiting until $\hat{t}^A = n+2$ and accepting

$\tilde{s}^{A,t+2}$ (step 4), and Opt (step 5). Note that when $t = \hat{t}^A - 2$, $A$ will not leave in the next time period, and thus $W$ will not consider this option.

The logic behind $A$'s strategy in the lemma above is as follows. $A$ prefers that $W$ leave. This is possible only if $Leave^W$ is not worse for $W$ than Opt. This situation is handled in step 1. In this case, $A$ rejects $W$'s strategy, waiting for it to leave in the next time period. Otherwise, $A$ will compare accepting $W$'s offer, $O^l$, $\tilde{s}^{A,t+1}$ in the next time period, leave now, or Opt now. As in previous cases, $A$ can be sure that $\tilde{s}^{A,t+1}$ will be accepted. Also, as in the previous lemma, if it has no preference between $O^l$ now and $\tilde{s}^{A,t+1}$ in the next period, and both are better than Opt, it will accept $O^l$ only if it is equal to $\tilde{s}^{A,t}$. Deviating will not increase its expected utility, and it is assumed that $A$ prefers to leave the resource free, as much as possible.

## 3.3 Possible agreements when $A$ prefers $\tilde{s}^{A,t+1}$ at $t+1$ to $\tilde{s}^{A,t+2}$ at $t+2$ (losing over time)

Before we continue to specify the strategies for $A$ and $W$, we will discuss more closely the case in which $A$'s expected utility from $\tilde{s}^{A,t}$ at period $t$ is higher than its expected utility of $\tilde{s}^{A,t+1}$ at period $t1$. We name this case "$A$ loses over time." As mentioned in Section 2.6.2, this is the case when $A$ has not yet performed $t^A_{min}$ time periods, it does not get paid for any period of the negotiation, and $\tilde{s}^{A,t}$ will not let it finish working for $t^A_{min}$. In this case, $W$ has more negotiation power than in the cases where $A$ does not lose over time. In particular, it may try to offer $A$ something that is better for $W$ than $\tilde{s}^{A,t}$. As can be seen in step 4 in $A$'s strategies of Lemma 6 and Lemma 9, in cases where Opt is better for $W$ than Leave, $A$, in time $t$, compares its offer with $\tilde{s}^{A,t+1}$, and if $\tilde{s}^{A,t}$ is better for $A$ than $\tilde{s}^{A,t+1}$, there is some freedom for $W$ here, in turn, regarding what to offer. The exact agreement for $W$ that will be better for $A$ than $\tilde{s}^{A,t+1}$ depends on the cost of holding the resource. We denote this agreement with $\bar{s}^{W,t}$. However, during time periods that are prior to those in which $W$ offers $\bar{s}^{W,t}$, $A$ will take this into consideration when it makes an offer. If it would like $W$ to accept an offer at time $t$ (since $A$ loses over time), it will need to offer $W$ an agreement that is better for $W$ than $\bar{s}^{W,t+1}$ and not just $\tilde{s}^{A,t}$. Otherwise, $W$ will reject the offer and suggest $\bar{s}^{W,t+1}$. Thus we define $\bar{s}^{W,t}$ backward from the first time that $A$ suggests $\tilde{s}^{A,t+1}$ to $W$. It is important to notice that when $A$ is losing over time, $\hat{t}^W \leq \hat{t}^A$. The backward definition of $\bar{s}^{W,t}$ will start from $\hat{t}$. We define $\hat{t}$ to be the latest time period such that $\hat{t} < \hat{t}^W$ and $U^W(Leave^W, \hat{t}) < U^W(Opt, \hat{t})$. In addition, we require that it be $A$'s turn to make an offer at time period $\hat{t} - 1$. It is easy to see that $\hat{t} < t^W_{neo}$, and that it is either $t^W_{neo} - 1$ or $t^W_{neo} - 2$, depending on whether $t^W_{neo}$ is even or odd.

**Definition 8 Base case ($t = \hat{t}$):** $\bar{s}^{W,\hat{t}} = \tilde{s}^{A,\hat{t}}$.

**$W$'s turn to respond ($t$ is odd):** *For any $t < \hat{t}$, if $t$ is odd then $\bar{s}^{W,n}$ is the best agreement for $A$ in time $t$ that is still better for $W$ than $\bar{s}^{W,t+1}$ at time $t+1$ and is not worse for $W$ than Opt at time $t$.*

**$A$'s turn to respond ($t$ is even):** *For any $t < \hat{t}$, if $t$ is even, then $\bar{s}^{W,n}$ is the best agreement for $W$ in time $t$ that is still better for $A$ than $\bar{s}^{W,t+1}$ at time $t+1$ and is not worse for $A$ than Opt at time $t$.*

Note that there are situations in which $\bar{s}^{W,t} = \tilde{s}^{A,t}$ for several time periods before $\hat{t}$. To demonstrate the way the values of $\bar{s}^{W,t}$ can be determined, we consider a revised version of the example of the robots on Mars.

**Example 4** Consider the case where the goal of RobotN is $< G1, 40, 80, 99, 4, 1002 >$, i.e., $t^A_{min} = 40$, $t^A_{max} = 80$ and its deadline is 99, but the other details of the situation are exactly as in Example 2. In particular, RobotE's goal is $< G2, 10, 20, 30, 4, 1002 >$, and the agents needs to pay 2 for holding the resource without doing productive work. We first need to determine $\tilde{s}^{A,0}$. This situation is similar to that of Example 3, since RobotE's goal hasn't changed and $s^0_A = 76$ is even larger than in the original situation. As in the original case, we need to compare $< 20, 10 >$ and $< 0, 0 >$. However, in this case, RobotN cannot perform the minimal required time needed for its goal in 20 minutes. This is since $t^A_{min} = 40$ and RobotN has worked only for 4 minutes before the negotiation. Thus its utility is only from future time periods after the agreement is implemented. Also, it will then be able to work only for 65 minutes if it gains access to the resource when $< 20, 10 >$ is implemented. However, it needs to pay for holding the resource for 20 minutes, since it is not productive during this time. So, $U^A(< 20, 10 >, 0) = 0.5 \cdot 65 \cdot 4 - 2 \cdot 20 = 90$. On the other hand, the agreement $< 0, 0 >$ will allow it to work for 80 minutes before its deadline and without paying for holding the resource; thus $U^A(< 0, 0 >, 0) = 0.5 \cdot 80 \cdot 4 = 160$. We can conclude that $\tilde{s}^{A,0} =< 0, 0 >$ and that it will remain $< 0, 0 >$ up to period 13 of the negotiation, at which time RobotE will not have had enough time to perform even its minimal requirements after opting out. During this time, RobotN's utility from $\tilde{s}^{A,t}$ decreases over time, since it needs to pay for holding the resource, i.e., $U^A(< 0, 0 >, 1) = 0.5 \cdot 80 \cdot 4 - 2 = 158$.

We will now consider the values of $t^W_{neo}$, $t^W_{ne}$, $\hat{t}^W$, and $\hat{t}^A$. $t^W_{neo} = 30 - 8 - 10 + 1 = 13$, $t^W_{ne} = 30 - 10 + 1 = 21$, and $\hat{t}^A = 76$. To compute $\hat{t}^W$, we must first compute $\tilde{s}^{A,13}$. Since at $t = 13$, $W$ has fewer future time periods than $t^W_{min}$ after opting out, $s^{13}_A = 67$ and $\tilde{s}^{A,13} =< 67, 0 >$. It is easy to see that $U^W(Leave^W, 13) \geq U^W(\tilde{s}^{A,13}, 13)$, and therefore $\hat{t}^W = 13$ and $\hat{t} = 12$. Thus, since $\tilde{s}^{A,12} =< 0, 0 >$, $\bar{s}^{W,12} =< 0, 0 >$. At time 12, RobotN can still do its 80 time periods, and since it needs to pay for holding the resource for 12 time periods, then $U^A(<0,0>,12) = 80 \cdot 4 \cdot 0.5 - 2 \cdot 12 = 136$. RobotE's utility from this is $U^W(<0,0>,12) = 18 \cdot 4 \cdot 0.5 - 2 \cdot 12 = 12$. To compute $\bar{s}^{W,11}$, we need to consider that in order for it to be better for $W$ than $<0,0>$, $W$ must obtain at least $t^W_{min} = 10$. However, the utility for RobotN from $<0,10>$ at time 12 is $73 \cdot 4 \cdot .5 - 2 \cdot 13 = 124$, which is lower than that of $<0,0>$ at time 12. Thus $\bar{s}^{W,11} =< 0,0 >$, and $\bar{s}^{W,10} = \tilde{s}^{A,10} =< 0,0 >$. This is the case, i.e., $\bar{s}^{W,t} = \tilde{s}^{A,t} =< 0,0 >$, until $t = 5$. Since at time 5 it is $A$'s turn to make an offer and $\bar{s}^{W,6} =< 0,0 >$, $\bar{s}^{W,5}$ is also equal to $<0,0>$, where $U^A(<0,0>,5) = 150$ and $U^W(<0,0>,5) = 30$. However, at time period 4, $W$ can offer something that is better for it than $<0,0>$. In particular, $\bar{s}^{W,4} =< 0,11 >$, where $U^A(<0,11>,4) = 152$ and $U^W(<0,11>,4) = 11 \cdot 4 + 9 \cdot 4 \cdot 0.5 - 2 \cdot 4 = 54$. Similarly, $\bar{s}^{W,3} =< 0,11 >$, $\bar{s}^{W,2} =< 0,13 >$, $\bar{s}^{W,1} =< 0,13 >$, and $\bar{s}^{W,0} =< 0,15 >$.

In this example, $\bar{s}^{W,t}$ enables RobotN to make use of its maximal future time periods, similar to its situation in $\tilde{s}^{A,t}$. However, in this example, if the payment for holding the resource is higher, or $p_A$ is lower, i.e., the losses of time are larger than the gain from additional working time periods, $A$'s share in $\bar{s}^{W,t}$ may decrease relative to $\tilde{s}^{A,t}$.

## 3.4 Specification of the subgame perfect equilibrium strategies

We are now ready to continue with the specification of the strategies that are in equilibrium.

First, we will present the strategies for any time period except the first one. For the last possible steps of the negotiations, we will combine the strategies for both $\hat{t}^W - 3$ and $\hat{t}^A - 3$, taking into consideration the differences in the strategies for $\hat{t}^W - 2$ and $\hat{t}^A - 2$. We denote by $\hat{t}$ the minimum of $\hat{t}^W$ and $\hat{t}^A$. Next, we will consider the special case of the first time period during which $W$ cannot opt out of the negotiation. Finally, we will combine all our results and present the subgame perfect equilibrium strategies for the entire negotiation. We will conclude this section with a short discussion of the results.

### 3.4.1 Negotiations starting at the second time period ($t = 1$)

In the next lemma, we specify the strategies for any time period $0 < t \leq \hat{t} - 3$.

**Lemma 10 (Strategies at $0 < t \leq \hat{t} - 3$)** Consider the negotiation at period $t$ such that $0 < t \leq \hat{t} - 3$, and suppose that the negotiation hasn't ended at period $t - 1$.

**$W$'s strategy:** If at $t$ it is $W$'s turn to respond to an offer, then it will use the following strategy:

1. **$Leave^W$ is not worse for $W$ than Opt in two time periods:**

   If $U^W(Leave^W, t+2) \geq U^W(Opt, t+2)$, then

   (a) If $U^W(< s_A^{t+1}, 0 >, t+1) \geq U^W(Opt, t)$, then

      i. If $U^W(s^{t+1}, t+1) \leq U^W(O^l, t)$, then say Yes.

      ii. Otherwise, say No and offer $\tilde{s}^{A,t+1}$.

   (b) If $U^W(Leave^W, t) < U^W(Opt, t)$, then if $U^W(Opt, t) \leq U^W(O^l, t)$, then say Yes. Otherwise, Opt.

   (c) Otherwise, if $U^W(Leave^W, t) \geq U^W(Opt, t)$, then if $U^W(Leave^W, t) < U^W(O^l, t)$, then say Yes. Otherwise, Leave.

2. **Waiting for $Leave^A$ is the best option:**

   Otherwise, if $U^A(Leave^A, t+1) \geq U^A(Opt, t+1) \geq U^A(\tilde{s}^{A,t+1}, t+1)$
   and $U^A(Leave^A, t+1) \geq U^A(\tilde{s}^{A,t+2}, t+2)$, then
   if $U^W(O^l, t) < U^A(Leave^A, t+1)$ then say No and offer $\tilde{s}^{A,t+2}$.

3. **Accepting $A$'s offer now is the best option:**

   Otherwise, if $U^W(O^l, t) \geq U^W(Opt, t)$ and
   [ $[U^A(\tilde{s}^{A,t+1}, t+1) > U^A(\tilde{s}^{A,t+2}, t+2)$ and $U^W(O^l, t) > U^W(\bar{s}^{W,t+1}, t+1)]$ OR
   $[U^A(\tilde{s}^{A,t+1}, t+1) = U^A(\tilde{s}^{A,t+2}, t+2)$ and $U^W(O^l, t) \geq U^W(\tilde{s}^{A,t+1}, t+1)]$ OR
   $[U^A(\tilde{s}^{A,t+1}, t+1) < U^A(\tilde{s}^{A,t+2}, t+2)$ and $U^W(O^l, t) \geq U^A(\tilde{s}^{A,t+2}, t+2)]$ ], then
   say Yes.

4. **$A$ prefers $\tilde{s}^{A,t+1}$ at $t+1$ than $\tilde{s}^{A,t+2}$ at $t+2$ (losing over time):**

   Otherwise, if $[U^A(\tilde{s}^{A,t+1}, t+1) > U^A(\tilde{s}^{A,t+2}, t+2)$ and $U^W(Opt, t) \leq U^W(\bar{s}^{W,t+1}, t+1)]$, then say No and offer $\bar{s}^{W,t+1}$.

5. **$A$ has no preference between $\tilde{s}^{A,t+1}$ at $t+1$ and $\tilde{s}^{A,t+2}$ at $t+2$:**

   Otherwise, if $U^A(\tilde{s}^{A,t+1}, t+1) = U^A(\tilde{s}^{A,t+2}, t+2)$ and $U^W(Opt, t) \leq U^W(\tilde{s}^{A,t+1}, t+1)$, then say No and offer $\tilde{s}^{A,t+1}$.

6. **$A$ prefers $\tilde{s}^{A,t+2}$ at $t+2$ than $\tilde{s}^{A,t+1}$ at $t+1$ (gaining over time):**

   Otherwise, if $[U^A(\tilde{s}^{A,t+1}, t+1) < U^A(\tilde{s}^{A,t+2}, t+2)$ and $U^W(Opt, t) \leq U^W(\tilde{s}^{A,t+2}, t+2)$, then say No and offer $\tilde{s}^{A,t+1}$.

7. **Opt is the best option:**

   Otherwise, Opt.

**$A$'s strategy:** If it is $A$'s turn to respond to an offer at period $t$, then it will use the following strategy:

1. $Leave^W$ **is not worse for** $W$ **than Opt:**

   If $U^W(Opt, t+1) \leq U^W(Leave^W, t+1)$, then say No and offer $< s_A^{t+1}, 0 >$.

2. $Leave^A$ **is the best option:**

   Otherwise, if $U^A(Leave^A, t) \geq U^A(Opt, t) \geq U^A(O^l, t)$
   and $U^A(Leave^A, t) \geq U^A(\tilde{s}^{A,t+1}, t+1)$, then Leave.

3. $A$ **prefers** $\tilde{s}^{A,t+1}$ **at** $t+1$ **than** $\tilde{s}^{A,t+2}$ **at** $t+2$ **(losing over time):**

   Otherwise, if $U^A(\tilde{s}^{A,t+1}, t+1) > U^A(\tilde{s}^{A,t+2}, t+2)$, then

   (a) If $U^A(O^l, t) > U^A(\bar{s}^{W,t+1}, t+1)$ and $U^A(O^l, t) \geq U^A(Opt, t)$, then say Yes.

   (b) Otherwise if $U^A(O^l, t) < U^A(\bar{s}^{W,t+1}, t+1)$ and $U^A(Opt, t) \leq U^A(\bar{s}^{W,t+1}, t+1)$, then say No and offer $\bar{s}^{W,t+1}$.

   (c) Otherwise, Opt.

4. $\tilde{s}^{A,t+1}$ **is the best option:**

   Otherwise, if $U^A(O^l, t) < U^A(\tilde{s}^{A,t+1}, t+1)$ and $U^A(Opt, t) \leq U^A(\tilde{s}^{A,t+1}, t+1)$, then say No and offer $\tilde{s}^{A,t+1}$.

5. $A$ **has no preference between** $\tilde{s}^{A,t+1}$ **and** $W$**'s offer:**

   Otherwise, if $U^A(O^l, t) = U^A(\tilde{s}^{A,t+1}, t+1)$, then

   (a) If $O^l = \tilde{s}^{A,t}$, then say Yes.

   (b) Otherwise, say No and offer $\tilde{s}^{A,t+1}$.

6. $W$**'s offer is the best option:**

   Otherwise, if $U^A(O^l, t) > U^A(\tilde{s}^{A,t+1}, t+1)$ and $U^A(O^l, t) \geq U^A(Opt, t)$, then say Yes.

7. **Opt is the best option:**

   Otherwise, Opt.

∎

The proof of the lemma is by backward induction on $t$. As in the previous lemmas, $W$'s position in the negotiation is weak when its expected utility from Leave is not higher than its expected utility from opting out. It cannot threaten $A$ with opting out and thus will not gain anything better than $< s_A^{t+1}, 0 >$ in the future. This situation is handled in step 1 of $W$'s strategy. Thus $W$ will compare Opting now, Leaving now, accepting $A$'s current offer, and reaching an agreement of $< s_A^{t+1}, 0 >$ in the next time period.

In step 2, $W$ checks whether there is a possibility that $A$ will leave in the next time period and whether it is worth it to wait for $A$ to leave.

In step 3, $W$ compares $A$'s offer with possible future outcomes. For $W$ to accept the offer, it needs to be the best option and also better than opting out. The possible future outcomes depend on whether $A$'s utility from $\tilde{s}^{A,t+2}$ at $t+2$ is higher than its utility from $\tilde{s}^{A,t+1}$ at $t+1$. If $A$'s utility from $\tilde{s}^{A,t+2}$ at $t+2$ is lower than its utility from $\tilde{s}^{A,t+1}$ at $t+1$, then $W$ will offer $\tilde{s}^{W,t}$, which will be accepted by $A$ in the next time period. If $A$'s utility from $\tilde{s}^{A,t+1}$ at $t+1$ is equal to its utility from $\tilde{s}^{A,t+2}$ at $t+2$, then $W$ will offer $\tilde{s}^{A,t+1}$, which will be accepted by $A$. If $A$'s utility from $\tilde{s}^{A,t+2}$ at $t+2$ is higher than its utility from $\tilde{s}^{A,t+1}$ at $t+1$, then the best $W$ can hope for in the future is $\tilde{s}^{A,t+2}$. If accepting $O^l$ now yields a higher utility than these possible outcomes, $W$ will accept it.

In steps 4-7, $W$ compares the above possible outcomes with Opt. If the relevant outcome is better than Opt out, then $W$ will say No and will make an offer. If Opt is better, $W$ will opt out.

The logic behind $A$'s strategy in the lemma above is as follows. As in the previous lemmas, if $Leave^W$ in the next time period is not worse for $W$ than Opt out in the next time period (step 1), then $A$ will wait until the next time period for $W$ to Leave. In step 2, $A$ considers leaving. Otherwise, if $A$'s utility from $\tilde{s}^{A,t+1}$ at $t+1$ is higher than its utility from $\tilde{s}^{A,t+2}$ at $t+2$, it will choose between accepting $W$'s offer (step 3a), $\tilde{s}^{W,t+1}$, in the next time period (step 3b), and Opt out now (step 3c). The rest of the strategy (steps 4-6) considers the situations in which $A$'s utility from $\tilde{s}^{A,t+1}$ at $t+1$ is not higher than its utility from $\tilde{s}^{A,t+2}$ at $t+2$. In these situations, $A$ will compare $A$'s offer, $\tilde{s}^{A,t+1}$ in the next time period and Opt. Note that if $A$ gains the same utility from $O^l$ and $\tilde{s}^{A,t+1}$, it will insist that $O^l = \tilde{s}^{A,t}$ according to the assumption "Outcome with the same utility".

### 3.4.2 Negotiation during the first time period

The first period of the negotiation is different from the others since $W$ cannot Opt, but can only make an offer or Leave the negotiation. In the next Lemma we specify $W$'s strategy in this period.

**Lemma 11 ($W$'s strategy during the first period ($t = 0$))** In the first period of the negotiation, $W$ will use the following strategy:

1. **$Leave^W$ in the second period ($t = 1$) is not worse for $W$ than Opt:**
   If $U^W(Opt, 1) \leq U^W(Leave^W, 1)$, then
   (a) If $U^W(Leave^W, 0) \geq U^W(< s_A^1, 0 >, 1)$, then Leave.
   (b) Otherwise, offer $\tilde{s}^{A,0}$.

2. **Waiting for $Leave^A$ is the best option:**
   Otherwise, if $U^A(Leave^A, 0) \geq U^A(Opt, 0) \geq U^A(\tilde{s}^{A,0}, 0)$
   and $U^A(Leave^A, 0) \geq U^A(\tilde{s}^{A,1}, 1)$, then
   offer $\tilde{s}^{A,0}$.

3. **$A$ prefers $\tilde{s}^{A,0}$ at 0 than $\tilde{s}^{A,1}$ at 1 (losing over time):**
   Otherwise, if $U^A(\tilde{s}^{A,0}, 0) > U^A(\tilde{s}^{A,1}, 1)$, then offer $\tilde{s}^{W,0}$.

4. **$A$ has no preference between $\tilde{s}^{A,0}$ now and $\tilde{s}^{A,1}$ in the next time period:**
   Otherwise, if $U^A(\tilde{s}^{A,0}, 0) = U^A(\tilde{s}^{A,1}, 1)$, then offer $\tilde{s}^{A,0}$.

5. **$A$ prefers $\tilde{s}^{A,t+2}$ at $t+2$ than $\tilde{s}^{A,t+1}$ at $t+1$ (gaining over time):**
   Otherwise, if $U^A(\tilde{s}^{A,0}, 0) < U^A(\tilde{s}^{A,1}, 1)$ and $U^W(\tilde{s}^{A,1}, 1) > U^W(Leave^W, 0)$, then offer $\tilde{s}^{A,0}$.

6. **Leave is the best option:**
   Otherwise, Leave.

∎

The logic behind the strategy in the previous lemma is that $A$ will accept an offer from $W$ only if it believes that $W$ can threaten it with Opt in the next time period.

The first step of the strategy treats the case in which it is better for $W$ to Leave in the second time period ($t = 1$) than to Opt. In this case, $W$'s threat to Opt is not credible. $A$, who prefers $Leave^W$ over any other option, will not accept any offer from $W$ and will offer $< s_A^1, 0 >$ in the next time period. This will allow $A$ to work for its maximal possible time periods. Thus, in this case, $W$ compares $< s_A^1, 0 >$ and Leave now, and decides either to Leave or to make some offer which it knows will be rejected.

The second step of the strategy considers the situation where $A$ will leave the negotiation when $W$ approaches it.

The third step of the strategy considers the situation where $A$'s utility from $\tilde{s}^{A,0}$ at 0 is higher than its utility from $\tilde{s}^{A,1}$ at 1. As explained above, $W$ can offer $\tilde{s}^{W,0}$, which will be accepted by $A$. If $A$ does not lose, but also does not gain over time (step 4), it will accept $\tilde{s}^{A,0}$. If $A$ gains over time (step 5), $W$'s offer will not be accepted. $A$ will offer $\tilde{s}^{A,1}$ in the next time period, which will be better for $W$ than Opt. Thus, $W$ is left with two options: accepting $\tilde{s}^{A,1}$ during the next time period, or Leave now. In the first case, it will offer $\tilde{s}^{A,0}$. If the second case occurs, it will Leave.

We can now combine the strategies presented in the lemmas above and specify the strategies that are in subgame-perfect equilibrium.

**Theorem 1 (Strategies)** The strategies below are in perfect equilibrium given our assumptions.

***A*'s strategy:**

1. **$A$ has finished working on its goal:**

   If $s_A^t = 0$, then Leave.

2. **$Leave^W$ is not worse for $W$ than Opt:**

   Otherwise, if $U^W(Opt, t+1) \leq U^W(Leave^W, t+1)$, then say No and offer $< s_A^{t+1}, 0 >$.

3. **$A$ prefers $(\tilde{s}^{A,t+1}, t+1)$ than $(\tilde{s}^{A,t+2}, t+2)$ (losing over time):**

   Otherwise, if $U^A(\tilde{s}^{A,t+1}, t+1) > U^A(\tilde{s}^{A,t+2}, t+2)$, then

   (a) If $U^A(O^l, t) > U^A(\bar{s}^{W,t+1}, t+1)$ and $U^A(O^l, t) \geq U^A(Opt, t)$, then say Yes.

   (b) Otherwise if $U^A(O^l, t) < U^A(\bar{s}^{W,t+1}, t+1)$ and $U^A(Opt, t) \leq U^A(\bar{s}^{W,t+1}, t+1)$, then say No and offer $\bar{s}^{W,t+1}$.

   (c) Otherwise, Opt.

4. **Accepting $W$'s offer is the best option:**

   Otherwise, if $U^A(O^l, t) > U^A(\tilde{s}^{A,t+1}, t+1)$ and $U^A(O^l, t) \geq U^A(Opt, t)$, then say Yes.

5. **$\tilde{s}^{A,t+1}$ in the next time period is the best option:**

   Otherwise, if $U^A(O^l, t) < U^A(\tilde{s}^{A,t+1}, t+1)$ and $U^A(Opt, t) \leq U^A(\tilde{s}^{A,t+1}, t+1)$, then say No and offer $\tilde{s}^{A,t+1}$.

6. **$A$ has no preference between accepting $W$'s offer now and $\tilde{s}^{A,t+1}$ in the next time period:**

   Otherwise, if $U^A(O^l, t) = U^A(\tilde{s}^{A,t+1}, t+1)$, then

   (a) If $O^l = \tilde{s}^{A,t}$, then say Yes.

   (b) Otherwise, say no and offer $\tilde{s}^{A,t+1}$.

7. **Opting out is the best option:**

   Otherwise, Opt.

***W*'s strategy:**

**$t = 0$:**

1. $Leave^W$ **in the second period** ($t = 1$) **is not worse for** $W$ **than Opt:**

   If $U^W(Opt, 1) \leq U^W(Leave^W, 1)$, then

   (a) $U^W(Leave^W, 0) \geq U^W(< s_A^1, 0 >, 1)$, then Leave.

   (b) Otherwise, offer $\tilde{s}^{A,0}$.

2. **Waiting for** $Leave^A$ **is the best option:**

   Otherwise, if $U^A(Leave^A, 0) \geq U^A(Opt, 0) \geq U^A(\tilde{s}^{A,0}, 0)$
   and $U^A(Leave^A, 0) \geq U^A(\tilde{s}^{A,1}, 1)$, then
   offer $\tilde{s}^{A,0}$.

3. $A$ **prefers** $(\tilde{s}^{A,0}, 0)$ **than** $(\tilde{s}^{A,1}, 1)$ **(losing over time):**

   Otherwise, if $U^A(\tilde{s}^{A,0}, 0) > U^A(\tilde{s}^{A,1}, 1)$, then offer $\tilde{s}^{W,0}$.

4. $A$ **has no preference between** $\tilde{s}^{A,0}$ **now and** $\tilde{s}^{A,1}$ **in the next time period:**

   Otherwise, if $U^A(\tilde{s}^{A,0}, 0) = U^A(\tilde{s}^{A,1}, 1)$, then offer $\tilde{s}^{A,0}$.

5. $A$ **prefers** $\tilde{s}^{A,t+2}$ **at** $t + 2$ **than** $\tilde{s}^{A,t+1}$ **at** $t + 1$ **(gaining over time):**

   Otherwise, if $U^A(\tilde{s}^{A,0}, 0) < U^A(\tilde{s}^{A,1}, 1)$ and $U^W(\tilde{s}^{A,1}, 1) > U^W(Leave^W, 0)$, then offer $\tilde{s}^{A,0}$.

6. **Leave is the best option:**

   Otherwise, Leave.

$t > 0$:

1. $Leave^W$ **is not worse for** $W$ **than Opt in two time periods:**

   If $U^W(Leave^W, t + 2) \geq U^W(Opt, t + 2)$, then

   (a) If $U^W(< s_A^{t+1}, 0 >, t + 1) \geq U^W(Opt, t)$, then

      i. If $U^W(< s_A^{t+1}, 0 >, t + 1) \leq U^W(O^l, t)$, then say Yes.

      ii. Otherwise, say No and offer $\tilde{s}^{A,t+1}$.

   (b) If $U^W(Leave^W, t) < U^W(Opt, t)$, then if $U^W(Opt, t) \leq U^W(O^l, t)$, then say Yes. Otherwise, Opt.

   (c) Otherwise, if $U^W(Leave^W, t) \geq U^W(Opt, t)$, then if $U^W(Leave^W, t) < U^W(O^l, t)$, then say Yes. Otherwise, Leave.

2. **Waiting for** $Leave^A$ **is the best option:**

   Otherwise, if $U^A(Leave^A, t + 1) \geq U^A(Opt, t + 1) \geq U^A(\tilde{s}^{A,t+1}, t + 1)$
   and $U^A(Leave^A, t + 1) \geq U^A(\tilde{s}^{A,t+2}, t + 2)$, then
   if $U^W(O^l, t) < U^A(Leave^A, t + 1)$ then say No and offer $\tilde{s}^{A,t+2}$.

3. **Accepting $A$'s offer now is the best option:**

   Otherwise, if $U^W(O^l,t) \geq U^W(Opt,t)$ and

   $[\ [U^A(\tilde{s}^{A,t+1},t+1) > U^A(\tilde{s}^{A,t+2},t+2)$ and $U^W(O^l,t) > U^W(\bar{s}^{W,t+1},t+1)]$

   OR

   $[U^A(\tilde{s}^{A,t+1},t+1) = U^A(\tilde{s}^{A,t+2},t+2)$ and $U^W(O^l,t) \geq U^W(\tilde{s}^{A,t+1},t+1)]$

   OR

   $[U^A(\tilde{s}^{A,t+1},t+1) < U^A(\tilde{s}^{A,t+2},t+2)$ and $U^W(O^l,t) \geq U^A(\tilde{s}^{A,t+2},t+2)]\ ]$,

   then say Yes.

4. **$A$ prefers $(\tilde{s}^{A,t+1},t+1)$ than $(\tilde{s}^{A,t+2},t+2)$ (losing over time)**:

   Otherwise, if $[\ [U^A(\tilde{s}^{A,t+1},t+1) > U^A(\tilde{s}^{A,t+2},t+2)$ and $U^W(Opt,t) \leq U^W(\bar{s}^{W,t+1},t+1)]$, then say No and offer $\bar{s}^{W,t+1}$.

5. **$A$ has no preference between $(\tilde{s}^{A,t+1},t+1)$ and $(\tilde{s}^{A,t+2},t+2)$:**

   Otherwise, if $U^A(\tilde{s}^{A,t+1},t+1) = U^A(\tilde{s}^{A,t+2},t+2)$ and $U^W(Opt,t) \leq U^W(\tilde{s}^{A,t+1},t+1)$, then say no and offer $\tilde{s}^{A,t+1}$.

6. **$A$ prefers $(\tilde{s}^{A,t+2},t+2)$ than $(\tilde{s}^{A,t+1},t+1)$ (gaining over time):**

   Otherwise, if $[U^A(\tilde{s}^{A,t+1},t+1) < U^A(\tilde{s}^{A,t+2},t+2)$ and $U^W(Opt,t) \leq U^W(\tilde{s}^{A,t+2},t+2)$, then say no and offer $\tilde{s}^{A,t+1}$.

7. **Opt is the best option:**

   Otherwise, Opt.

**Proof:**

The proof is clear from the previous lemmas. ∎

It it important to discuss the ways the negotiations may end according to the above theorem:

**$W$ will Leave without starting a negotiation process:** If, in the second period of the negotiation, $W$ does not have enough time to do $t^W_{min}$ before its deadline after opting out, i.e., $U^W(Opt,1) \leq U^W(Leave^W,1)$, then it considers leaving because $A$ will reject any offer and make a counteroffer $< s^1_A, 1 >$. If $W$ prefers Leaving in period 0 over such an agreement, it will Leave.

Another situation in which $W$ may Leave before even starting the negotiations is when $A$ gains over time, i.e., if $U^A(\tilde{s}^{A,0},0) < U^A(\tilde{s}^{A,1},1)$ and $W$ prefers Leaving over $(\tilde{s}^{A,1},1)$.

**$A$ will Leave when $W$ approaches it:** If $A$'s utility from $Leave^A$ at the first time period (0) is not lower than its utility from Opt at period 0 and from $\tilde{s}^{A,1}$ at period 1, it will leave in the first period of the negotiations. This is the case when $A$ hasn't worked for $t^A_{min}$ before $W$ approaches it, and it does not have enough time to finish working for $t^A_{min}$ before it needs to give $W$ the resource so that $W$ will be able to work for $t^W_{min}$ before $W$'s deadline, and it does not have enough time to work on its goal after opting out.

**An agreement will be reached in the first time period:** If in period 1, $W$ prefers Opt over Leave and $A$'s utility from $\tilde{s}^{A,0}$ period 0 is not lower that its utility from $\tilde{s}^{A,1}$ at period 1, then an agreement will be reached in the first period of the negotiation.

**An agreement will be reached in the second time period:** If, in period 1, $W$ prefers Opt over Leave, $A$'s utility $\tilde{s}^{A,1}$ at period 1 is higher than its utility from $\tilde{s}^{A,0}$ at 0 and $W$ prefers $< \tilde{s}^{A,1}, 1 >$ over Leaving before starting the negotiation, then the negotiation will end during the second period of the negotiation.

## 3.5 Examples

We will consider several examples of negotiation to demonstrate the cases where $A$ prefers $(\tilde{s}^{A,t+1}, t+1)$ to $(\tilde{s}^{A,t+2}, t+2)$ (losing over time), $A$ prefers $(\tilde{s}^{A,t+2}, t+2)$ to $(\tilde{s}^{A,t+1}, t+1)$ (gaining over time), or $A$ has no preference between $(\tilde{s}^{A,t+1}, t+1)$ and $(\tilde{s}^{A,t+2}, t+2)$. We will also consider an example where $W$ Leaves before even starting the negotiation and an example where $A$ leaves when $W$ approaches him.

### 3.5.1 $A$ has no preference between $(\tilde{s}^{A,t+1}, t+1)$ and $(\tilde{s}^{A,t+2}, t+2)$

We will demonstrate the negotiation in this case using the original Mars example.

**Example 5** We return to the example of the robots on Mars (Examples 2 and 3). Recall that RobotN's goal is $< G1, 15, 70, 79, 4, 1002 >$, i.e., $t^A_{min} = 15$, $t^A_{max} = 70$ and its deadline is 79, and that of RobotE is $< G2, 10, 20, 30, 4, 1002 >$. As calculated in Example 3, $\tilde{s}^{A,0} =< 20, 10 >$, $\tilde{s}^{A,1} =< 19, 10 >$ and $\tilde{s}^{A,2} =< 18, 10 >$, etc., until period $t^W_{neo} = 13$ where $\tilde{s}^{A,13} =< 53, 0 >$. Recall that RobotN's utility from $\tilde{s}^{A,t}$ does not change over time until $t = 13$. RobotE's utility from $\tilde{s}^{A,t}$ decreases over time. However, its utility from Opting out also decreases over time. As was computed in Example 2, $U^W(Opt, G2, 0, 0) = 40$, $U^W(Opt, G2, 0, 1) = 38$ and $U^W(Opt, G2, 0, 2) = 36$. Furthermore, $U^W(Opt, G2, 0, 3) = 19 \cdot 4 \cdot .5 - 2 \cdot 3 = 32$; $U^W(Opt, G2, 0, 4) = 18 \cdot 4 \cdot .5 - 2 \cdot 4 = 28$; $U^W(Opt, G2, 0, 12) = 10 \cdot 4 \cdot .5 - 2 \cdot 12 = -4$; $U^W(Opt, G2, 0, 13) = -2 \cdot 13 = -26$; $U^W(Leave^W, G2, 0, 0) = 0$; $U^W(Leave^W, G2, 0, 1) = -2$.

Since $t^W_{neo} = 13$, and $U^A(\tilde{s}^{A,0}, 0) = U^A(\tilde{s}^{A,1}, 1)$, this is equivalent to case 4 of $W$'s strategy for $t = 0$ in Theorem 1. Thus RobotE will offer $\tilde{s}^{A,0} =< 20, 10 >$, which will be accepted by RobotN.

### 3.5.2 $A$'s utility from $(\tilde{s}^{A,t}, t)$ is not lower than that from $(\tilde{s}^{A,t+1}, t+1)$

We will demonstrate this case using a modification of the example of the robots on Mars.

**Example 6** We return to the example of the robots on Mars (Example 2) but assume that RobotN works on goal $< G1, 13, 27, 30, 4, 1002 >$, $t^A_{min} = 13$, $t^A_{max} = 27$ and the deadline is 30. RobotE needs to work on $< G2, 2, 12, 20, 4, 1002 >$, i.e., $t^W_{min} = 2$, $t^W_{max} = 12$ and the deadline is 20. The other details remain as in Example 2, e.g., $done^A = 4$.

RobotN would like to work for an additional 23 minutes to satisfy its maximal possible requirement; i.e., $s^A_0 = 23$. However, this will prevent RobotE from satisfying even its minimal requirement. We shall now compute $\tilde{s}^{A,t}$. In the first time period, $W$ can accomplish its goal completely by working for $t^W_{max}$ time periods after opting out, but after $s^A_0$ periods $W$ will not have enough time before its deadline to accomplish $t^W_{max}$ periods. We need to compare RobotN's utilities from the following: $< 10, 2 >$, $< 14, 6 >$ and $< 0, 0 >$. $U^A(< 10, 2 >, 0) = 10 \cdot 4 + 13 \cdot 4 \cdot .5 = 66$; $U^A(< 14, 6 >, 0) = 14 \cdot 4 + 6 \cdot 4 \cdot .5 = 68$ and $U^A(< 0, 0 >, 0) = 26 \cdot 4 \cdot .5 = 52$. Thus $\tilde{s}^{A,0} =< 14, 6 >$. RobotE's utility from this agreement is: $U^W(< 14, 6 >, 0) = 6 \cdot 4 = 24$ and its utility from Opt is $U^W(Opt, 0) = 12 \cdot 4 \cdot .5 = 24$.

At time period 1, RobotE does not have enough time to complete its maximal requirement after opting out before its deadline. In this period, $s^1_A = 22$. We need to compare $< 8, 0 >$, whose utility at period 1 to RobotN is 64, $< 10, 2 >$, whose utility is 68; $< 13, 5 >$, whose utility is 70; and $< 0, 0 >$, whose utility is 50. Thus $\tilde{s}^{A,1} =< 13, 5 >$. RobotE's utility from $< 13, 5 >$ at period 1 is: $U^W(< 13, 5 >, 1) = 4 \cdot 5 + 1 \cdot 4 \cdot 0.5 - 2 = 20$. Its utility from opting out is $U^W(Opt, 1) = 11 \cdot 4 \cdot .5 - 2 = 20$.

In the next time period, $t = 2$ and $s^2_A = 21$, and again we will compare $< 8, 0 >$, $< 10, 2 >$, $< 13, 5 >$, and $< 0, 0 >$. RobotN achieves maximum utility from $< 13, 5 >$, and it is 72 (i.e., RobotN's utility continues to increase). Thus $\tilde{s}^{A,2} =< 13, 5 >$. RobotE's utility from $\tilde{s}^{A,2}$ is 16, and so its utility from Opt is also 16. Note that in this time period RobotE's utility from Leave is much lower than from Opt. Thus, according to Theorem 1, it offers $\tilde{s}^{A,0}$, but RobotN, which prefers $\tilde{s}^{A,1}$, will say no and will offer $\tilde{s}^{A,1}$. RobotE will accept the offer and the negotiation will end in the second round of the negotiation.

### 3.5.3 $A$ prefers $(\tilde{s}^{A,t+1}, t+1)$ to $(\tilde{s}^{A,t+2}, t+2)$ (losing over time)

$A$ prefers $(\tilde{s}^{A,t+1}, t+1)$ to $(\tilde{s}^{A,t+2}, t+2)$ when it can't work for at least $t^A_{min}$ and make $W$ an offer that will prevent it from opting out. This case happens when $W$'s deadline expires (so it cannot work $t^W_{min}$) *before* $A$ can work for (at least) $t^A_{min}$ time periods on its goal, i.e. $dl^W - t^W_{min} < t^A_{min} - done^A$. We have already considered this situation in Example 4. We will present the resolution of the negotiation in this case in the following example.

**Example 7** We return to the example of the robots on Mars when the situation is exactly as in Example 4. That is, the goal of RobotN is $< G1, 40, 80, 99, 4, 1002 >$, and the goal of RobotE is $< G2, 10, 20, 30, 4, 1002 >$. We have shown in Example 4 that $A$'s utility from $\tilde{s}^{A,t}$ at period $t$ is higher than its utility from $\tilde{s}^{A,t+1}$ at period $t+1$ and found that $\bar{s}^{W,0} =< 0, 15 >$.

Since $t^W_{neo} = 13$, $U^W(Opt, 1) > U^W(Leave^W, 1)$, and since $A$ loses over time, we are in case 3 of $W$'s strategy, when $t = 0$, in Theorem 1. Thus RobotE will offer $< 0, 15 >$, which $A$ will accept according to step 3a of its strategy.

In summary, the negotiation will end in the first period of the negotiation with an agreement.

### 3.5.4 $W$ Leaves before starting the negotiation

There are situations in which, regardless of whether $A$ gains or loses over time, $W$ will Leave the negotiation. We demonstrate this case in the following example.

**Example 8** We modify the example of the robots on Mars. Suppose that RobotN's goal is $< G1, 15, 70, 79, 4, 1002 >$, as in Example 2, but RobotE's deadline is earlier: $< G2, 10, 20, 15, 4, 1002 >$. The rest of the details of the example are as in Example 2, e.g., $q = 8$. In this case, in the first time period RobotE does not have enough time to work for 10 time periods after opting out before the deadline. Thus RobotE's utility from Opt and $Leave^W$ in the first time period is equal to zero. RobotN would like to work for an additional 66 time periods, will not accept any offer, and will offer $< 66, 0 >$ to RobotE in the second time period of the negotiation. RobotE prefers to Leave rather than to accept such an offer. Thus RobotE will Leave before starting a negotiation process.

### 3.5.5 $A$ leaves when $W$ approaches it

**Example 9** We modify the example of the robots on Mars. Suppose that RobotN's goal is $< G1, 10, 12, 12, 4, 1002 >$, i.e., $t^A_{min} = 10$, $t^A_{max} = 12$ and its deadline is 12, and it has worked on its goal for 5 time periods (i.e., $done^A = 5$) before RobotE starts the negotiation.

That is, when $W$ starts the negotiation, there is only 7 time periods left until $A$'s deadline. RobotE's goal is $< G2, 10, 12, 14, 4, 1002 >$ and $q = 2$.

If $A$ will continue working on its goal and finish its minimal number of time periods, i.e., work for an additional 5 time periods, $W$ will not be able to complete its own minimal time periods since its deadline is in 14 time periods, and it needs to work for at least 10 time periods. Thus, $\tilde{s}^{A,0} =< 0, 0 >$.

In addition, if $A$ interrupts its work, it will not be able to work for its minimal number of time periods (i.e., for 10 periods) because its deadline is in 7 time periods when the negotiation begins. Thus $A$'s utility from opting out, leaving, and the agreement $\tilde{s}^{A,0} =< 0, 0 >$ at time period 0 are the same. Its utility from $\tilde{s}^{A,1} =< 0, 0 >$ at time period 1 is even lower since it needs to pay for using the resource.

However, $W$ prefers to opt out than to leave at time period 1, since after opting out it will need to wait for 2 time periods and then it will still have enough time to work on its goal for the minimal number of time periods before its deadline.

Thus, when $W$ will approach $A$ with the offer $\tilde{s}^{A,0} =< 0, 0 >$, $A$ will leave, knowing that otherwise $W$ will opt out in the next time period and given its preferences of leaving now to $W$'s opting out in the next time period.

# 4 Simulation Results

This section presents the simulation results of the protocol and the strategies discussed in the previous sections. We will see how different variables, such as the cost of opting out, and the goals' deadlines affect the performance of the algorithms. We have also compared our algorithms to a well-known algorithm, Earliest-Deadline-First (EDF), which has been shown to achieve good scheduling results.

Several simplifying assumptions are required to apply the formal model and to make a reasonable implementation feasible. First, it is assumed that communication is foolproof and that the message delay time is known to all agents. Second, in order to carry out coordination activities, agents share a global clock reference. However, we would like to emphasize that the system is distributed in such a way that there is no one process which manages the others; each process acts independently.

The experiments were conducted with a system comprising three agents: $A_1$, $A_2$, and $A_3$. Each agent has 50 goals, $G_1, \ldots, G_{50}$, to accomplish. The agents' utility functions satisfied our assumptions and their details are presented in [Sch96]. Such a scenario was tested for 10 runs, to guarantee that all results presented were significant to the 0.05 level or below ($p < 0.05$).

| | Field | Description | Values |
|---|---|---|---|
| Goal | $t_{min}$ | minimum time periods needed for working in order to be paid for a goal | [1,10] |
| | $t_{max}$ | maximum time periods needed for working on a goal | [$t_{min}$+1,20] |
| | $dl$ | deadline to accomplish a goal | [$t_{max}$+1,$t_{max}$+10] |
| | m | payment per time period | [4,10] |
| Resource | c | cost per time unit when using the resource | [1,3] |
| | l | cost per time unit when holding the resource | [0,c-1] |
| General | q | time periods needed for repairing the resource after opting out | 1,3,6 |

Table 2: Values of the parameters used in simulation.

Table 2 shows the values of all parameters used in the experiments. Note that $q$ was tested for different values. All other parameters were created using a uniform distribution.

## 4.1 Metrics

The agents' utility function has been used as the main metric for evaluation and comparison. We also count the number of negotiation sessions and the number of goals that were not completed successfully. The following are formal definitions of the metrics we used in our experiments.

**Definition 9 Utility score:**
*The utility gained when running a session, divided by the maximum utility that could be gained when each agent has all needed resources, is the utility score. The ratio will be represented as a percentage.*

Since the EDF algorithm tries to maximize the number of goals that were completed successfully, we add the following metric to our tests.

**Definition 10 Abandoned goals:**
*The percentage of goals that the agents are forced to abandon due to deadlines are called* Abandoned goals*.*

The following metric is an attribute of the environment, which specifies the load of the system: how many times the agents needed to use a strategy to solve a conflict for sharing resources. We chose environments with heavy loads, because we believe that in such environments our model is applicable.

| q | Utility score | | | | Opting Outs | Abandoned goals | Nego. sessions |
|---|---|---|---|---|---|---|---|
| | A1 | A2 | Average | St.Dev. | out of 30 | out of 30 | |
| 1 | 88% | 94% | 91% | 5% | 1 | 9.6 | 43% |
| 3 | 94% | 87% | 90% | 5% | 1 | 9.6 | 43% |
| 6 | 84% | 93% | 89% | 5% | 0 | 9.6 | 43% |

Table 3: Experimental results with different values of $q$.

**Definition 11 Negotiation/Alternations:**
*The number of times a conflict arises between two agents over the usage of a common resource is called* Negotiation/Alternations. *The conflict is solved either by negotiation or by alternation, depending on the model used.*

## 4.2 Results and discussion

Recall that our model is designed to maximize the utility function of *each agent* rather than the utility of the agents as a group. Therefore, both the utility score of each agent and the average score were considered.

We ran the simulations with different values of the $q$ parameter, that is, the time delay due to opting out. As Table 3 shows, our model gives a fair share of the resources to all agents. This is even though agents that play the role of $A$ in the negotiations usually obtain a higher utility. However, since the agents have an equal probability of playing the role of $A$ and $W$ in the simulations, fair distribution is obtained.

In addition, we can see that agents chose to opt out very rarely. This happened only in cases of incomplete information that we did not consider in our theoretical work. When an agent Opts out during negotiation, both the attached agent, that is currently using the resource, and the agent that opted out must wait $q$ time periods until the resource is repaired. However, under the EDF model, when preemption occurs no payment is made by either side: neither by the attached agent nor by the new agent that gained access to the resource.

Another interesting development is the evolution of the agents' achievements under different opting out constraints. As shown in Table 3, as $q$ increases, the agents' average utility score decreases slightly. However, the frequency of opting outs decreases, meaning that the agents avoid opting out when it hurts their achievements. The difference between the agents' scores, i.e., A1 vs. A2, is insignificant. It is a result of applying goals with different attributes.

In the second set of experiments, we used the same data sets for two algorithms: our negotiation model and the EDF algorithm. As shown in Table 4, our model did as well as EDF, with small differences in the standard deviation.

| Metric | Negotiation (q=1) | | EDF | | Conclusion |
|---|---|---|---|---|---|
| | Average | St.Dev. | Average | St.Dev. | |
| Utility score | 91% | 5.2% | 91% | 3.5% | Same |
| Abandoned goals | 9.6 | 3.1 | 8.4 | 2.6 | Same |
| Nego. / Alternation | 21.2 | 4.5 | 15.5 | 1.7 | Less in EDF |

Table 4: Experimental results, comparing Negotiation and EDF.

| Index | Description | Utility score (q=1) | Changes in Metrics |
|---|---|---|---|
| 1 | $t^W_{min} = 0$ | 91% | Number of opting outs increases. |
| 2 | $t^W_{min} = t^W_{max}$ | 91% | Number of abandoned goals decreases. |
| 3 | $P_t(t) = 0$ | 91% | Remain the same. |

Table 5: Experimental results of studying the effect of different parameters on the negotiation results.

In addition to its achievements with respect to the utility score metric, our algorithm outperforms EDF in other aspects. First, the EDF algorithm can be used only in cases where the central agent has complete information on all agents competing for the resource. Another advantage of our model over EDF is its flexibility. Each agent may opt out and reject its opponents' offers. Such an option does not exist in EDF which forces the agents to accept decisions. Furthermore, we must be sure that any new agent that might be created in the future will obey the EDF scheduler.

In our model, which uses negotiation as a way of sharing resources, each agent accepts only the communication protocol, without any need for additional rules concerning the schedule itself. We believe that a system with fewer rules is more flexible and thus would be able to meet more future needs, as they arise.

The third set of experiments was designed to study the effect of different parameters on the negotiation results (table 5).[9] We first considered the case in which the minimum time period ($t^i_{min}$) is zero, meaning that the agent gets paid from the first working time period. The average utility score remains the same as before, but opting out increased. Next, we studied a case in which each goal requires a specific amount of time to be achieved, and the agent gets paid only when it accomplishes maximum periods (i.e., $t^i_{min} = t^i_{max}$). Again, the average utility score remains the same, but the number of abandoned goals increases. The last modification in our model was to eliminate the effect of negotiation cost. The results show that there was no difference between the two scenarios.

[9]See [Sch96] for a formal analysis of this issue.

## 5 Related work

Our research applies modified game theory techniques to a distributed artificial intelligence (DAI) environment. In this section we discuss relevant work on using negotiation in DAI environments similar to ours. For related work in game-theory, resource allocation, and scheduling in real-time systems see [Sch96].

Bond and Gasser [BG88] divide DAI research into two basic classes: *Distributed Problem Solving (DPS)* and *Multi-Agent Systems (MA)*. The main difference between these two categories is in the motivation of the agents to cooperate. Whereas in *DPS*, all agents act to achieve a common interest, in *MA*, agents act from individual (and possibly contradictory) interests. Research in MA is concerned with coordinating intelligent behavior among a collection of autonomous (possibly heterogeneous) intelligent (possibly pre-existing) agents. In MA, there is no global control and there are no globally shared goals or success criteria. Thus, a possibility exists for *real competition* among the agents. Our work falls in the Multi-Agent (MA) category, since our agents are assumed to be non-benevolent, and each acts to maximize its own utility, rather than the group's utility. Negotiation has been used in MA environments as a means of achieving mutually beneficial agreements.[10]

Rosenschein and Zlotkin [RZ94] have identified three distinct domains where negotiation is applicable and have found a different strategy for each domain: (i) Tasks-Oriented Domain: finding ways in which agents can negotiate and come to agreement, and allocating their tasks in a way that is to everyone's benefit; (ii) State-Oriented Domain: finding actions which change the state of the "world" and serve the agents' goals; and (iii) Worth-Oriented Domain: same as the above, but, in this domain, the decision is taken according to the maximum utility the agents gain from the states. Our model belongs to the worth-oriented domain. Rosenschein and Zlotkin do not discuss the issue of resource allocation, but focus on the issue of performing the agents' goals. They assume that the agents' goals can be divided into several distinct subgoals. States in which only some of the subgoals are achieved are rated lower than states in which they are all achieved. Mapping our problem to their framework, we can assume that the subgoals are working in a given time period. However, time plays no explicit role in the agent's utility functions of Rosenschein and Zlotkin [RZ94], and therefore, their strategies cannot be used in our model.

Sycara [Syc90, Syc87] presented a model of negotiation that combines case-based reasoning and optimization of multi-attribute utilities. In her work, agents try to influence the goals and intentions of their opponents. We do not model the agents' intentions in our model. We assume that the agents' goals are fixed before the negotiation starts. Thus,

[10]Research that applies negotiation to DPS environments includes [CML88, DL89, MLB92, KG94].

the agents in our model do not argue about the goals, but rather exchange offers. This is possible because we assume that there is complete information and each agent can compute the utility of both agents from a given agreement.

In [KL95], Kraus and Lehmann developed an automated Diplomacy player that negotiates and plays well in actual games against human players. The Diplomacy environment is more complex than the environments that we consider in this paper, (e.g. more agents, more possible actions, simultaneous actions). Thus, finding strategies that are in equilibrium is not possible and in [KL95] heuristics that were developed based on informal models (e.g., [Rai82, FU81, Dru77, Kar70, Joh93, Hal93]) were applied.

Sierra et al. [SFJ97] present a model of negotiation for autonomous agents to reach agreements about the provision of service by one agent to another. Similar to [KL95], the model defines a range of strategies and tactics, distilled from intuition about good behavioral practice in human negotiation, that agents can employ to generate offers and evaluate proposals. Their agents are able to negotiate in more complex environment than ours, but no formal justification for their strategies is provided.

Zeng and Sycara [ZS98] consider negotiation in a marketing environment with a learning process in which the buyer and the seller update their beliefs about the opponent's reservation price using the Bayesian rule. In the current paper, we consider situations of complete information and thus learning is not needed. However, the utility functions of our agents are more complex, and finding stable strategies depends on the specification of the agents' goals. Sandholm and Lesser [SL95] discuss issues, such as levels of commitment, that arise in automated negotiation among self-interested agents whose rationality is bounded by computational complexity. These issues are presented in the context of iterative task allocation negotiations, while we consider negotiation on resource allocation. The study of these issues in the context of our environment is left for future work.

Sandholm and Vulkan [SV99] consider situations of bilateral negotiation where agents have firm deadlines that are private information. The agents negotiate on splitting a dollar and are allowed to make and accept offers in any order in continuous time. Sandholm and Vulkan proved that agents cannot agree to a nontrivial split because offers signal sufficient bargaining power weakness (early deadline) so that the recipient would never accept. We consider more complex scenarios, where the agents need to reach an agreement on two attributes and their utility depends on the exact specification of their goal and their status with respect to the resource. We also assume that the agents can opt out of the negotiations. These assumptions lead to non-trivial agreements. In addition, while Sandholm and Vulkan focus on the problems resulting from the agents' incomplete information, in this paper, we assume that the agents have complete information. We are currently working on extending our results to situations where the agents do not know the details of the other's goal (e.g., its deadline).

Kraus, Wilkenfeld, and Zlotkin [KWZ95] have proposed a strategic model of negotiation that takes the passage of time during the negotiation process itself into account. Their model considers situations characterized by complete, as well as incomplete, information and situations in which some agents lose over time, while others gain over time. Our research is strongly based on this work. We extended the former model in several directions in order for the model to be applicable to more complex environments.

- Two attributes in an agreement: In previous work, there was only one attribute to the negotiation. In this paper, an agreement consists of two parts.

- Goals have deadlines: In this paper, the agents' goals have deadlines. Thus, the possible beneficial agreements are limited and the utility functions are much more complex. It is common to assume that goals have deadlines [But97], and if we want the negotiations method to be applicable to resource allocation, we need to allow this assumption.

The results of [KWZ95] are not applicable in environments with the above features. The main question is whether, when considering more complex environments, the important property, that the negotiation should end without delay, remains valid. We were able to identify stable strategies for the agents in the environments with two attributes and deadlines. Even though the strategies are more complex than in the strategies identified in [KWZ95], the negotiation ends after the first time period.

Other work in the DAI community dealing with the resource allocation problem includes, for example, [CML88, KL89], which present a multistage negotiation protocol that is useful for cooperatively resolving resource allocation conflicts arising in distributed networks of semi-autonomous problem solving nodes. A key element in their solution is the ability to detect subgoal interactions in a distributed environment and reason about their impact. The interactions between goals in our domain is very simple, and the aim of the negotiations between our agents is to resolve conflicts between opponents, and not to solve local conflicts between cooperative agents.

Combinatorial auction has been proposed for resource allocation when the value of some resources to an agent depends upon which other resources it obtains (e.g., [LOS99, FLBS99, San02].)

Lesser et al. [LPD88] address tradeoffs in resource allocation and real-time performance, and develop a mechanism for resource allocation based on the criticality of tasks. They focus on the planning issue, while we assume that no planning is needed for the satisfaction of the goals, but rather the agents should solve their conflict on the usage of the resource. In more recent work [HVM+01, HLVW02] they extended their model to deal with dynamic soft-real time constraints which requires the negotiation protocol to adapt to the available time left.

Kornfeld and Hewitt [KH81] propose resource allocation using specialist "sponsor" agents. Their main attempt is the development of a language for the interactions between the agents. The agents in our environment exchange only messages that include simple agreements.

Chandrasekn [Cha81] proposes resource allocation via resource pricing. Sengupta and Findler [SF92] discuss dynamic scoping and constrained lattice-like structures for distributed resource allocation in multi-agent environments. Chavez et al. [CMM97] developed a market-based control system, named Challenger, that performs distributed resource allocation (in particular, allocation of CPU time.) Whereas all these works are applicable to DPS environments, we consider MA environments.

# 6 Conclusion and Future Work

This work is concerned with how automated agents can be designed to interact effectively in order to share resources. A strategic model of negotiation has been proposed as a way of reaching mutual benefit, while avoiding costly and time-consuming interactions which might increase the overhead of coordination. That is, we have provided a model in which two agents can avoid spending too much time negotiating an agreement and will therefore be better able to adhere to a timetable for satisfying their goals.

There are a number of directions which we think are interesting and are worth further investigation. The main extensions that are needed are (i) negotiations when the agents have incomplete information about their opponents' utility function; (ii) negotiations between more than two agents. These are enhancements to our theoretical model, and hence require theoretical support before implementing them in the simulator. We emphasize that given our simulation tools, implementing any strategies specified by the theoretical results is straightforward.

# References

[Ber98] K. Berns. Walking machine catalogue. http://www.fzi.de/ipt/WMC/preface/walking_machines_katalog.html, 1998.

[BG88] A. H. Bond and L. Gasser. An analysis of problems and research in DAI. In A. H. Bond and L. Gasser, editors, *Readings in Distributed Artificial Intelligence*, pages 3–35. Morgan Kaufmann Publishers, Inc., San Mateo, California, 1988.

[But97] Giorgio C. Buttazzo. *Hard Real-Time Computing Systems.* Kluwer Academic Publishers, Boston, MA, 1997.

[Cha81] B. Chandrasekn. Natural and social system metaphors for distributed problem solving: Introduction to the issue. *IEEE Transaction on Systems Man and Cybernetics*, 11(1):1–5, 1981.

[CML88] S. E. Conry, R. A. Meyer, and V. R. Lesser. Multistage negotiation in distributed planning. In A. H. Bond and L. Gasser, editors, *Readings in Distributed Artificial Intelligence*, pages 367–384. Morgan Kaufmann Publishers, Inc., San Mateo, California, 1988.

[CMM97] A. Chavez, A. Moukas, and P. Maes. Challenger: A multi-agent system for distributed resource allocation. In W. Lewis Johnson and Barbara Hayes-Roth, editors, *Proceedings of the 1st International Conference on Autonomous Agents*, pages 323–331, New York, 1997. ACM Press.

[DL89] E. H. Durfee and Victor R. Lesser. Negotiating Task Decomposition and Allocation Using Partial Global Planning. In L. Gasser and M. N. Huhns, editors, *Distributed Artificial Intelligence, Volume II*, pages 229–244. Pitman/Morgan Kaufmann, London, 1989.

[Dru77] D. Druckman. *Negotiations.* Sage Publications, London, 1977.

[FLBS99] Y. Fujishima, K. Leyton-Brown, and Y. Shoham. Taming the computational complexity of combinatorial auctions: Optimal and approximate approaches. In Dean Thomas, editor, *Proceedings of the 16th International Joint Conference on Artificial Intelligence (IJCAI-99-Vol1)*, pages 548–553, S.F., July 31–August 6 1999. Morgan Kaufmann Publishers.

[FU81] R. Fisher and W. Ury. *Getting To Yes: Negotiating Agreement Without Giving In.* Houghton Mifflin, Boston, 1981.

[GCE+97] M. P. Golombek, R. A. Cook, T. Economou, W. M. Folkner, A. F. Haldemann, P. H. Kallemeyn, J. M. Knudsen, R. M. Manning, H. J. Moore, T. J. Parker, R. Rieder, J. T. Schofield, P. H. Smith, and R. M. Vaughan. Overview of the mars pathfinder mission and assessment of landing site predictions. *Science*, 278(5):1743–1748, December 1997.

[Hal93] Lavinia Hall, editor. *Negotiation: Strategies for Mutual Gain.* Sage, Beverly Hills, 1993.

[HLVW02] Bryan Horling, Victor Lesser, Regis Vincent, and Thomas Wagner. The soft real-time agent control architecture. *Proceedings of the AAAI/KDD/UAI-2002 Joint Workshop on Real-Time Decision Support and Diagnosis Systems*, April 2002. Also available as UMass Computer Science Tech Report 02-14.

[HVM+01] B. Horling, R. Vincent, R. Mailler, J. Shen, R. Becker, K. Rawlins, and V. Lesser. Distributed sensor network for real time tracking. In *Proceedings of the 5th International Conference on Autonomous Agents*, pages 417–424, Montreal, June 2001. ACM Press.

[Joh93] R. Johnson. *Negotiation basics.* Sage, Newbury Park, 1993.

[Kar70] C.L Karrass. *The Negotiating Game: How to Get What You Want.* Thomas Crowell Company, NY, 1970.

[KG94] T. Khedro and M. Genesereth. Progressive negotiation for resolving conflicts among distributed heterogeneous cooperating agents. In *Proc. of AAAI94*, pages 381–386, 1994.

[KH81] W. Kornfeld and C. Hewitt. The scientific community metaphor. *IEEE Transaction on Systems Man and Cybernetics*, 11(1):24–33, 1981.

[KL89] K. Kuwabara and V. Lesser. Extended protocol for multistage negotiation. In *Proc. of the Ninth Workshop on Distributed Artificiall Intelligence*, pages 129–161, Rosario, Washington, September 1989.

[KL95] S. Kraus and D. Lehmann. Designing and building a negotiating automated agent. *Computational Intelligence*, 11(1):132–171, 1995.

[KWZ95] S. Kraus, J. Wilkenfeld, and G. Zlotkin. Multiagent negotiation under time constraints. *Artificial Intelligence*, 75(2):297–345, 1995.

[LOS99] Daniel Lehmann, Liadan Ita O'Callaghan, and Yoav Shoham. Truth revelation in rapid, approximately efficient combinatorial auctions. In *Proceedings of the ACM*

*Conference on Electronic Commerce (EC'99)*, pages 96–102, N.Y., November 3–5 1999. ACM Press.

[LPD88] V. R. Lesser, J. Pavlin, and E. H. Durfee. Approximate processing in real time problem solving. *AI Magazine*, 9(1):49–61, 1988.

[LR57] R. D. Luce and H. Raiffa. *Games and Decisions.* John Wiley and Sons, New York, 1957.

[MLB92] T. Moehlman, V. Lesser, and B. Buteau. Decentralized negotiation: An approach to the distributed planning problem. *Group Decision and Negotiation*, 2:161–191, 1992.

[Nas53] J. F. Nash. Two-person cooperative games. *Econometrica*, 21:128–140, 1953.

[OR90] M. J. Osborne and A. Rubinstein. *Bargaining and Markets.* Academic Press Inc., San Diego, California, 1990.

[OR94] M. J. Osborne and A. Rubinstein. *A Course in Game Theory.* MIT Press, Cambridge, Massachusetts, 1994.

[Rai82] H. Raiffa. *The Art and Science of Negotiation.* Harvard University Press, Cambridge, MA, 1982.

[Rub82] A. Rubinstein. Perfect equilibrium in a bargaining model. *Econometrica*, 50(1):97–109, 1982.

[RZ94] J. S. Rosenschein and G. Zlotkin. *Rules of Encounter: Designing Conventions for Automated Negotiation Among Computers.* MIT Press, Boston, 1994.

[San02] Tuomas Sandholm. Algorithm for optimal winner determination in combinatorial auctions. *Artificial Intelligence*, 135(1-2):1–54, 2002.

[Sch96] O. Schechter. Sharing resources through negotiation in multi-agent environments. Master's thesis, Bar-Ilan University, Ramat-Gan, Israel, November 1996.

[SF92] U. Sengupta and N. V. Findler. Multi-agent planning and collaboration in dynamic resource allocation. In J. Hendler, editor, *Artificial Intelligence Planning Systems: Proceedings of the First International Conference (AIPS 92)*, pages 305–306, College Park, Maryland, USA, June 1992. Morgan Kaufmann.

[SFJ97] C. Sierra, P. Faratin, and N. Jennings. A service-oriented negotiation model between autonomous agents. In *Proc. 8th European Workshop on Modeling Autonomous Agents in a Multi-Agent World (MAAMAW-97)*, pages 17–35, Ronneby, Sweden, 1997.

[SL95] T. W. Sandholm and V. R. Lesser. Issues in automated negotiation and electronic commerce: Extending the contract net framework. In *First International Conference on Multiagent Systems (ICMAS-95)*, pages 328–335, San Fransisco, 1995.

[SV99] T. W. Sandholm and N. Vulkan. Bargainig with deadlines. In *Proc. of AAAI99*, Orlando, Florida, 1999.

[Syc87] K. P. Sycara. *Resolving Adversarial Conflicts: An Approach to Integrating Case-Based and Analytic Methods.* PhD thesis, School of Information and Computer Science, Georgia Institute of Technology, 1987.

[Syc90] K. P. Sycara. Persuasive argumentation in negotiation. *Theory and Decision*, 28:203–242, 1990.

[Tir88] Jean Tirole. *The Theory of Industrial Organization.* The MIT Press, Cambridge, MA, 1988.

[ZS98] D. Zeng and K. Sycara. Bayesian learning in negotiation. *International Journal of Human-Computer Studies*, 48:125–141, 1998.